MATHEMATICS SAMPLE PAPER # 2

ANSWER AND SOLUTIONS

SECTION-A

1. Option (b)
-1

2. Option (a)
(3, 1)

3. Option (b)
$k \le 4$

4. Option (c)
(8, 20)

5. Option (a)
$60°$

6. Option (d)
9 units

7. Option (a)
7.8

8. Option (b)
162

9. Option (a)

$$-\frac{9}{4}$$

10. Option (a)
0

11. Option (d)
3

12. Option (c)
25

13. Option (d)
16.8 cm

14. Option (c)

$$\frac{5}{4}$$

15. Option (c)
4

16. Option (d)
17.5

17. Option (b)
$\tan 30°$

18. Option (a)

$\sqrt{119}$

19. Option (d)

Assertion (A) is false but Reason (R) is true.

20. Option (c)

Assertion (A) is true but Reason (R) is false.

SECTION-B

21. Number divisible by 8 between 200 and 500 are 208, 216, 224, ...496 which forms an A.P.

$\therefore$ First term (a) = 208, common difference (d) = 8

n^{th} term of an A.P. is $a_n = a + (n - 1)d$

$496 = 208 + (n - 1)8$

$\Rightarrow \quad 288 = (n - 1)8$

$\Rightarrow \quad n - 1 = 36$

$\Rightarrow \quad n = 37$

OR

Here, $a = 16$, $\ell = 128$

$$S_n = \frac{n}{2}(a + \ell)$$

$$= \frac{8}{2}(16 + 128)$$

$= 4 \times 144$

$= 576$

22. Total possible outcomes = $6 \times 6 = 36$

Favourable outcomes are {(1, 6), (2, 3), (3, 2), (6, 1)} i.e. 4 in number.

$\therefore$ P(getting the product 6) = $\frac{4}{36} = \frac{1}{9}$

23. If height is 40 cm

circumference of base of cylinder = 22 cm

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{7}{2} \text{ cm}$$

24. Any number which ends in zero must have at least 2 and 5 as prime factors.

$6 = 2 \times 3$

$6^n = (2 \times 3)^n$

$= 2^n \times 3^n$

Hence, prime factor of 6 are 2 and 3

Thus, 6^n can never end with digit 0.

OR

$90 = 2 \times 3^2 \times 5$

$144 = 2^4 \times 3^2$

$HCF = 2 \times 3^2 = 18$

$LCM = 2^4 \times 3^2 \times 5 = 720$

25. Let P(x, y) is equidistant from A(–5, 3) and B(7, 2)

AP = BP

$\Rightarrow \sqrt{((x+5)^2 + (y-3)^2)} = \sqrt{((x-7)^2 + (y-2)^2)}$

$\Rightarrow x^2 + 10x + 25 + y^2 - 6y + 9$

$= x^2 - 14x + 49 + y^2 - 4y + 4$

$10x - 6y + 34 = -14x - 4y + 53$

$10x + 14x - 6y + 4y = 53 - 34$

$24x - 2y = 19$

$24x - 2y - 19 = 0$

is the required relation.

SECTION-C

26. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10 \text{ cm}^2$

$= 220 \text{ cm}^2$

Curved surface area of a hemisphere = $2\pi r^2$

Curved surface area of both hemispheres

$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \text{ cm}^2$

$= 154 \text{ cm}^2$

Total surface area of the remaining solid

= (Curved surface area of cylinder + curved surface area of 2 hemispheres)

$= (220 + 154) \text{ cm}^2 = 374 \text{ cm}^2$.

OR

Given : d = 24 m, h = 3.5 m

r = 12 m

Volume of rice $= \frac{1}{3}\pi\, 12^2 \times 3.5 = 528 \text{ m}^3$

Canvas cloth required to cover heap

$= \pi r\ell$ .... (a)

$\ell = \sqrt{12^2 + 3.5^2} = 12.50$

From (a)

Cloth required $= \frac{22}{7} \times 12 \times 12.5 = 471.43 \text{ m}^2$

27.

Salary (₹ in thousand)	Number of Persons	c.f.
5 – 10	49	49
10 – 15	133	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280

$n = 280, \frac{n}{2} = 140$

So, median class is 10 – 15

$\ell = 10, cf = 49, f = 133, h = 5$

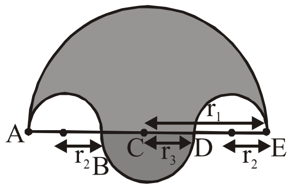
Median $= \ell + \frac{\frac{n}{2} - cf}{f} \times h$

$= 10 + \frac{140 - 49}{133} \times 5$

$= 10 + 3.42$

$= 13.42$

28. Let the radii of the largest semicircle, the smallest semicircle and the circle with diameter BD be r_1, r_2 and r_3 respectively.



Given, AE = 14 cm $\Rightarrow r_1 = 7$ cm

and DE = AB = 3.5 cm $\therefore r_2 = \frac{3.5}{2}$ cm

$r_3 = r_1 - 2r_2 = 7 - 2 \times \frac{3.5}{2} = 7 - 3.5 = 3.5$ cm

Area of the shaded region = Area of semicircle with radius r_1 + Area of semicircle with radius r_3 – 2 × Area of semicircle with radius r_2

$= \frac{1}{2}\pi(r_1)^2 + \frac{1}{2}\pi(r_3)^2 - 2\times\frac{1}{2}\pi(r_2)^2$

$= \frac{1}{2}\pi\{(r_1)^2 + (r_3)^2 - 2(r_2)^2\}$

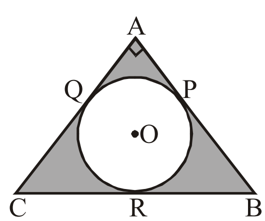
$= \frac{1}{2}\times\frac{22}{7}\left\{(7)^2 + (3.5)^2 - 2\left(\frac{3.5}{2}\right)^2\right\}$

$= \frac{11}{7}\left\{49 + 12.25 - \frac{12.25}{2}\right\}$

$= \frac{11}{7}(49 + 6.125)$

$= \frac{11}{7} \times 55.125 = 86.625 \text{ cm}^2$

OR



Given, AB = 6 cm and BC = 10 cm

By pythagoras theorem, in ΔABC, we get

$AC^2 = BC^2 - AB^2 = (10)^2 - (6)^2 = 64$

$\Rightarrow$ AC = 8 cm

Let the radius of the incircle be r.

Let the circle touch side AB at P, side AC at Q and side BC at R.

Join OP, OQ and OR.

We know that the radius from the centre of the circle is perpendicular to the tangent through the point of contact.

$\therefore OP \perp AB, OQ \perp AC$ and $OR \perp BC$

Also, the tangents drawn from an external point to the circle are equal.

$\therefore$ AP = AQ, BP = BR, CR = CQ

Now, in quadrilateral

AQ = AP and $\angle AQO = \angle APO = \angle PAQ = 90°$

OPAQ is a square.

$\therefore$ OP = AQ = AP = OQ = r

$\therefore$ PB = 6 – r $\Rightarrow$ BR = 6 – r

CQ = 8 – r $\Rightarrow$ CR = 8 – r

Now, BC = BR + CR

$\Rightarrow$ 10 = 6 – r + 8 – r $\Rightarrow$ 10 = 14 – 2r

$\Rightarrow$ r = 2 cm

Now, area of shaded region

= Area of ΔABC – Area of circle

$= \frac{1}{2} \times AB \times AC - \pi r^2 = \frac{1}{2} \times (8) \times (6) - 3.14(b)^2$

$= 24 - 12.56 = 11.44 \text{ cm}^2$

29. Sum of all the prizes = Rs.700

Let the first prize = a

$\therefore$ 2^{nd} prize = (a – 20)

3^{rd} prize = (a – 40)

4^{th} prize = (a – 60)

Thus, we have, first term = a

Common difference = –20

Sum of 7 terms $S_7 = 700$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 700 = \frac{7}{2}[2(a) + (7-1)\times(-20)]$

$\Rightarrow 700 = \frac{7}{2}[2a + (6 \times -20)]$

$\Rightarrow 700 \times \frac{2}{7} = 2a - 120$

$\Rightarrow 200 = 2a - 120 \Rightarrow 2a = 200 + 120 = 320$

$\Rightarrow a = \frac{320}{2} = 160$

Thus, the values of the seven prizes are Rs.160, Rs.(160 – 20), Rs.(160 – 40), Rs.(160 – 80), Rs.(160 – 100) and Rs.(160 – 120) = Rs.160, Rs.140, Rs.120, Rs.100, Rs.80, Rs.60 and Rs.40

30. LHS = (1 + cotA – cosecA)(1 + tanA + secA)

$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$

$= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$

$= \frac{(\sin A+\cos A)^2-1}{\sin A\cos A} = \frac{1+2\sin A\cos A-1}{\sin A\cos A}$

$= \frac{2\sin A\cos A}{\sin A\cos A}$

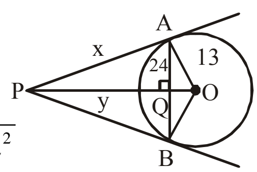
= 2

= RHS

Hence proved.

31. BQ = 12 cm,

OB = 13 cm



$\therefore$ OQ = $\sqrt{13^2-12^2}$

= $\sqrt{169-144}$ = $\sqrt{25}$

OQ = 5 cm

Let PQ = y and PA = x

In Δ POA : $x^2 + 13^2 = (y + 5)^2$

$x^2 + 169 = y^2 + 10y + 25$

: $x^2 - y^2 + 169 - 25 = 10y$ ... (a)

In ΔPQA : $x^2 = 12^2 + y^2$

$x^2 - y^2 = 144$ ... (b)

Put (b) in (a) 144 + 169 – 25 = 10y

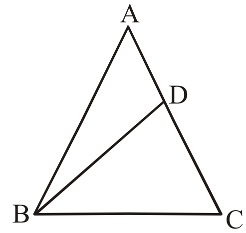
10y = 288 ⇒ y = 28.8

PA = x = $\sqrt{144+(28.8)^2}$ = $\sqrt{973.44}$

= 31.2 cm

SECTION-D

32.



We are given that

$BC^2 = AC \times CD$

$\Rightarrow \frac{BC}{CD} = \frac{AC}{BC}$ ......(a)

In ΔABC and ΔBDC, we have

$\frac{AC}{BC} = \frac{BC}{CD}$ [Using (a)]

and ∠BCA = ∠DCB [Each = ∠C of ΔABC]

⇒ ΔABC ~ ΔBDC [By SAS similarity]

$\Rightarrow \frac{AC}{BD} = \frac{BC}{CD}$ [∵ AB = AC is given]

$\Rightarrow \frac{AC}{BC} = \frac{BD}{CD}$

i.e., $\frac{BD}{CD} = \frac{AC}{BC}$ ....(b)

From (a) and (b), we have

$\frac{BD}{CD} = \frac{BC}{CD}$ [∵ Each = $\frac{AC}{BC}$]

⇒ BD = BC.

Hence proved

33. Let the usual speed of the train be x km/h

$\frac{300}{x} - \frac{300}{x+5} = 2$

$\Rightarrow x^2 + 5x - 750 = 0$

$\Rightarrow (x + 30)(x - 25) = 0$

$\Rightarrow x = -30, 25$

∴ Usual speed of the train = 25 km/h

OR

$$\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{b+a}{ab}$$

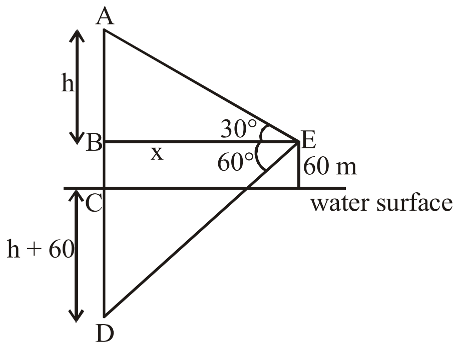
$\Rightarrow -ab = x^2 + (a + b)x$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow (x + a)(x + b) = 0$

$\Rightarrow x = -a, -b$

34.



In ΔABE,

$$\frac{h}{x} = \tan 30°$$

$$\Rightarrow x = h\sqrt{3}$$

In ΔBDE,

$$\frac{h+60+60}{x} = \tan 60°$$

$h + 120 = x\sqrt{3}$

$h + 120 = h\sqrt{3}\times\sqrt{3}$

$2h = 120$

$h = 60$

$\therefore$ height of cloud from surface of water

$= (60 + 60)m = 120$ m

35. Two solutions of each linear equation

$x + 3y = 6$ ...(i)

and $2x - 3y = 12$ ...(ii)

are given below.

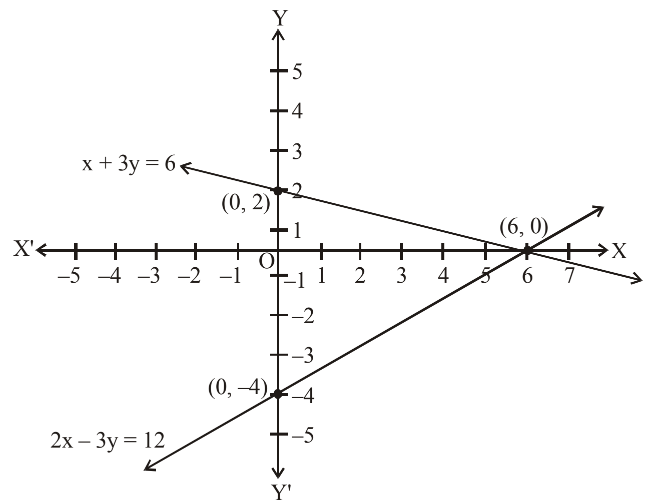
(i)

x	6	0
y	0	2

(ii)

x	6	0
y	0	– 4

The graphical representation of the given pair of linear equations is as follows :



Thus, the coordinates of point where the line $x + 3y = 6$ intersects the y-axis at (0, 2) and the line $2x - 3y = 12$ intersects the y-axis at (0, – 4).

OR

Let the fraction be $\frac{x}{y}$.

According to question

$\therefore x + y = 2x + 4 \Rightarrow x = y - 4$ ....(i)

Also, $\frac{x+3}{y+3} = \frac{2}{3}$

$$\Rightarrow \frac{y-4+3}{y+3} = \frac{2}{3}$$

$$\Rightarrow \frac{y-1}{y+3} = \frac{2}{3}$$

$\Rightarrow 3y - 3 = 2y + 6 \Rightarrow y = 9$

Substituting the value of y in (i), we get

$x = 5$

Thus, the required fraction is $\frac{5}{9}$.

SECTION-E

36. (i) Coordinates of $S = \left(\frac{-3+3}{2}, \frac{4+4}{2}\right) = (0, 4)$

(ii) Coordinates of $T = \left(\frac{3-2}{2}, \frac{4-1}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

(iii) Centroid of $\Delta PQR = \left(\frac{-3+3-2}{3}, \frac{4+4-1}{3}\right)$

$= \left(\frac{-2}{3}, \frac{7}{3}\right)$

Coordinates of

$U = \left(\frac{-3-2}{2}, \frac{4-1}{2}\right) = \left(\frac{-5}{2}, \frac{3}{2}\right)$

OR

Coordinates of Centroid of ΔSTU

$$= \left(\frac{0-\frac{5}{2}+\frac{1}{2}}{3}, \frac{4+\frac{3}{2}+\frac{3}{2}}{3}\right) = \left(\frac{-2}{3}, \frac{7}{3}\right)$$

37. (i) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

(ii) Minimum number of books

= LCM (32, 36)

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$36 = 2 \times 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$

(iii) $HCF = \frac{32 \times 36}{288} = 4$

OR

$p = ab^2$

$= a \times b \times b$

$q = a^2b = a \times a \times b$

$LCM(p, q) = a \times a \times b \times b = a^2b^2$

$HCF(p, q) = a \times b = ab$

38. (i) $\angle ORP = \angle OQP = 90°$

In quadrilateral ROQP

$\angle P + \angle O + \angle ORP + \angle OQP = 360°$

$\Rightarrow 30° + \angle O + 90° + 90° = 360°$

$\angle O = 180° - 30°$

$\angle ROQ = 150°$

(ii) In ΔORQ

$\Rightarrow \angle QOR + \angle QRO + \angle RQO = 180°$

$\Rightarrow 150° + \angle QRO + \angle QRO = 180°$

[$\because \angle QOR = \angle RQO$]

$\Rightarrow 2\angle QRO = 180° - 150° = 30°$

$\Rightarrow \angle QRO = 15°$

(iii) In ΔORQ

$OQ = OR$ [Radii of same circle]

$\angle OQR = \angle ORQ$ [Angle opposite to equal sides are equal]

$\angle OQR + \angle OQR + 150° = 180°$

$\angle OQR = 15°$

$\angle RQP = 90° - 15° = 75°$

OR

$\angle SRQ = \angle RQP = 75°$[Corresponding angles]

$\angle SRO = 75° - 15° = 60°$