JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON WEDNESDAY 21st JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If the domain of the function

$f(x) = \cos^{-1}\left(\frac{2x-5}{11-3x}\right) + \sin^{-1}(2x^2 - 3x + 1)$ is the interval $[\alpha, \beta]$, then $\alpha + 2\beta$ is equal to :

(1) 1 (2) 3

(3) 5 (4) 2

Ans. (2)

Sol. $f(x) = \cos^{-1}\left(\frac{2x-5}{11-3x}\right) + \sin^{-1}(2x^2 - 3x + 1)$

$$-1 \le \frac{2x-5}{11-3x} \le 1$$

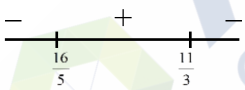
$$-1 \le 2x^2 - 3x + 1 \le 1$$

$$2x^2 - 3x + 2 \ge 0,\ 2x^2 - 3x \le 0$$

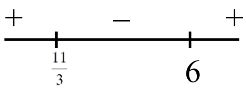
$$x \in \left[0, \frac{3}{2}\right] \quad .....(i)$$

$$\frac{2x-5}{11-3x} + 1 \ge 0 \qquad \frac{2x-5}{11-3x} - 1 \le 0$$

$$\frac{2x-5+11-3x}{11-3x} \ge 0 \qquad \frac{5x-16}{11-3x} \le 0$$



$$\frac{6-x}{11-3x} \ge 0$$



$$x \in \left(-\infty, \frac{16}{5}\right] \cup \left(\frac{11}{3}, \infty\right)$$

$$x \in \left(-\infty, \frac{11}{3}\right) \cup [6, \infty)$$

intersection

$$x \in \left(-\infty, \frac{16}{5}\right] \cup [6, \infty) \quad ...(ii)$$

Intersection of (i) & (ii) $x \in \left[0, \frac{3}{2}\right]$

$$\alpha = 0,\ \beta = \frac{3}{2} \Rightarrow \alpha + 2\beta = 3$$

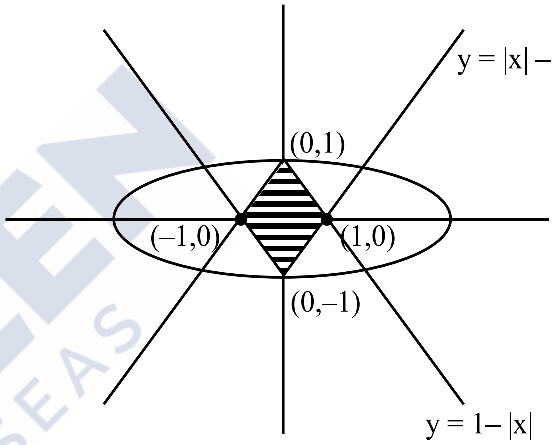
2. The area of the region, inside the ellipse $x^2 + 4y^2 = 4$ and outside the region bounded by the curves $y = |x| - 1$ and $y = 1 - |x|$, is :

(1) $2(\pi - 1)$ (2) $2\pi - \frac{1}{2}$

(3) $3(\pi - 1)$ (4) $2\pi - 1$

Ans. (1)

Sol.



Required area = area of ellipse – shaded area

$$= \pi \times 2 \times 1 - 4\left(\frac{1}{2} \times 1 \times 1\right) = 2\pi - 2$$

3. The number of relations, defined on the set $\{a, b, c, d\}$, which are both reflexive and symmetric, is equal to:

(1) 256 (2) 16

(3) 1024 (4) 64

Ans. (4)

Sol. Number of relation which are reply and sym. both

$= 1^4 \times 2^6 = 64$

(a, a)	(a, b)	(a, c)	(a, d)
(b, a)	(b, b)	(b, c)	(b, d)
(c, a)	(c, b)	(c, c)	(c, d)
(d, a)	(d, b)	(d, c)	(d, d)

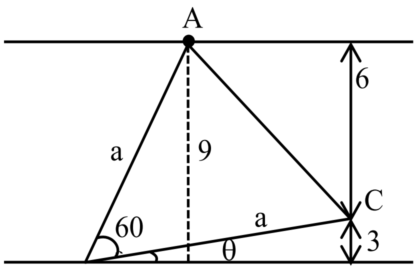
4. Let a point A lie between the parallel lines L_1 and L_2 such that its distances from L_1 and L_2 are 6 and 3 units, respectively. Then the area (in sq. units) of the equilateral triangle ABC, where the points B and C lie on the lines L_1 and L_2 respectively, is :

(1) $15\sqrt{6}$ (2) 27

(3) $21\sqrt{3}$ (4) $12\sqrt{2}$

Ans. **(3)**

Sol.



$$\sin\theta = \frac{3}{a}$$

$$\sin(60^\circ + \theta) = \frac{9}{a}$$

$$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{9}{a}$$

$$\sqrt{3}\sqrt{1-\frac{9}{a^2}} + \frac{3}{a} = \frac{18}{a}$$

$$a = \sqrt{84}$$

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 84 = 21\sqrt{3}$$

5. Let $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c} = \vec{b}$. If $\vec{c}.(\hat{i}+\hat{j}+\hat{k}) = 4$, then $|\vec{a}+\vec{c}|^2$ is equal to :

(1) 33 (2) 30

(3) 35 (4) 27

Ans. **(4)**

Sol. $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{a}\times\vec{c} = \vec{b} \Rightarrow (2c_3 - 2c_2)\,\hat{i} + (c_3 + 2c_1)\,\hat{j} - (c_2 + 2c_1)\,\hat{k} = 8\hat{i} + 7\hat{j} - 3\hat{k}$

$2c_3 - 2c_2 = 8,\ c_3 + 2c_1 = 7,\ c_2 + 2c_1 = 3$

$(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}).(\hat{i}+\hat{j}+\hat{k}) = 4$

$\Rightarrow c_1 + c_2 + c_3 = 4,\ c_1 = 2,\ c_2 = -1,\ c_3 = 3$

$|\vec{a}+\vec{c}|^2 = |\hat{i}+\hat{j}+5\hat{k}|^2 = 27$

6. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms such that $a_2.a_3.a_4 = 64$ and $a_1 + a_3 + a_5 = \frac{813}{7}$.

Then $a_3 + a_5 + a_7$ is equal to :

(1) 3256 (2) 3252

(3) 3244 (4) 3248

Ans. **(2)**

Sol. $ar \,.\, ar^2 \,.\, ar^3 = 64$

$a^3r^6 = 64 \quad \Rightarrow \quad ar^2 = 4$

$a + ar^2 + ar^4 = \frac{813}{7}$

$r^2 = 28$

$ar^2 + ar^4 + ar^6 = ?$

$ar^2(1 + r^2 + r^4) = 4(1 + 28 + 784) = 3252$

7. Let $\vec{c}$ and $\vec{d}$ be vectors such that $|\vec{c}+\vec{d}| = \sqrt{29}$ and $\vec{c}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{d}$. If $\lambda_1, \lambda_2 (\lambda_1 > \lambda_2)$ are the possible values of $(\vec{c}+\vec{d}).(-7\hat{i}+2\hat{j}+3\hat{k})$, then the equation

$$K^2x^2 + (K^2 - 5K + \lambda_1)xy + \left(3K + \frac{\lambda_2}{2}\right)y^2 - 8x + 12y + \lambda_2 = 0$$

represents a circle, for k equal to :

(1) 4 (2) 1

(3) –1 (4) 2

Ans. **(2)**

Sol. $|\vec{c}+\vec{d}| = \sqrt{29}$

$\vec{c}+\vec{d} = \lambda\left((2\hat{i}+3\hat{j}+4\hat{k}\right)$

$\lambda = \pm 1$

$\lambda(-14+6+12) = 4\lambda$, $\lambda_1 = 4$, $\lambda_2 = -4$

$k^2x^2 + (k^2 - 5k + 4)\,xy + (3k - 2)\,y^2 - 8x + 12y - 4 = 0$

is circle

$k^2 - 5k + 4 = 0 \Rightarrow k = 1, 4$

$k^2 = 3k - 2 \quad \Rightarrow k = 1, 2$

$k = 1$

8. Let y = y(x) be the solution curve of the differential equation $(1+x^2)dy + (y - \tan^{-1}x)dx = 0$, $y(0) = 1$. Then the value of y(1) is :

(1) $\dfrac{2}{e^{\frac{\pi}{4}}}+\dfrac{\pi}{4}-1$ (2) $\dfrac{2}{e^{\frac{\pi}{4}}}-\dfrac{\pi}{4}-1$

(3) $\dfrac{4}{e^{\frac{\pi}{4}}}+\dfrac{\pi}{2}-1$ (4) $\dfrac{4}{e^{\frac{\pi}{4}}}-\dfrac{\pi}{2}-1$

Ans. (1)

Sol. $\dfrac{dy}{dx}+\dfrac{y}{x^2+1}=\dfrac{\tan^{-1}x}{x^2+1}$

$\text{I.f.} = e^{\tan^{-1}x}$

$y \times e^{\tan^{-1}x} = \int e^{\tan^{-1}x}.\dfrac{\tan^{-1}x}{1+x^2}dx$

$y \times e^{\tan^{-1}x} = \tan^{-1}x\,(e^{\tan^{-1}x}) - e^{\tan^{-1}x} + c$

$y(o) = 1 \Rightarrow c = 2$

$y(1) = \dfrac{2}{e^{\pi/4}}+\dfrac{\pi}{4}-1$

9. The number of strictly increasing functions f from the set {1, 2, 3, 4, 5, 6} to the set (1, 2, 3,…,9) such that $f(i) \neq i$ for $1 \leq i \leq 6$, is equal to :

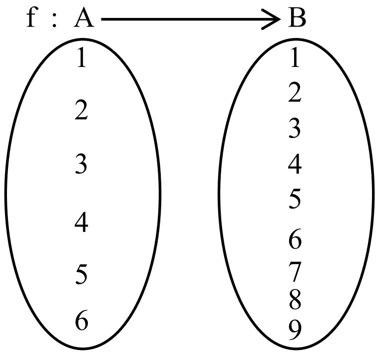
(1) 21 (2) 27

(3) 22 (4) 28

Ans. (4)

Sol. $f(i) \neq i$, f(x) is strictly increasing function $f: A \to B$, where A {1,2,3…….6}

B {1,2,3,……9}, then number of function $f: A \to B$ is equal to



$f(i) \neq i$ Case – i $f(1) = 2 \Rightarrow {}^7C_5 = 21$

Case- ii $f(1) = 3 \Rightarrow {}^6C_5 = 6$

Case- iii $f(1) = 4 \Rightarrow {}^5C_5 = 1$

No of function A to B = 21 + 6 + 1 = 28

10. Let $f: R \to (0, \infty)$ be a twice differentiable function such that f(3) = 18, f'(3) = 0 and f''(3) = 4. Then $\lim\limits_{x\to1}\left(\log_e\left(\dfrac{f(2+x)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)$ is equal to :

(1) 1 (2) 9

(3) 2 (4) 18

Ans. (3)

Sol. Let $T = \lim\limits_{x\to1}\left(\dfrac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}$; 1^∞ form

$\Rightarrow T = e^{\lim\limits_{x\to1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)-f(3)}{f(3)}\right)}$

$\Rightarrow T = e^{\lim\limits_{x\to1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)-f(3)}{18}\right)}$

$\Rightarrow T = e^{\lim\limits_{x\to1}\left(\frac{f(x+2)-f(3)}{(x-1)^2}\right)\frac{0}{0}\text{ form}}$ apply L' pital

$\Rightarrow T = e^{\lim\limits_{x\to1}\frac{f'(x+2)}{2(x-1)};\frac{0}{0}\text{ form}}$ apply L' pital

$\Rightarrow T = e^{\lim\limits_{x\to1}\frac{f''(x+2)}{2}}$; $= e^{\frac{4}{2}} = e^2$

$\Rightarrow \log_e(T) = 2$

11. Let the foci of hyperbola coincide with the foci of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If the eccentricity of the hyperbola is 5, then the length of its latus rectum is:

(1) 12 (2) 16

(3) $\frac{96}{\sqrt{5}}$ (4) $24\sqrt{5}$

Ans. (3)

Sol. Let e_1 be eccentricity of ellipse

$$\Rightarrow e_1=\sqrt{1-\frac{16}{36}}=\sqrt{1-\frac{4}{9}}=\frac{\sqrt{5}}{3}$$

So $ae_1=6.\frac{\sqrt{5}}{3}=2\sqrt{5}$

Now $H: \frac{x^2}{p^2}-\frac{y^2}{q^2}=1$

$p.e=ae_1$

$p.5=2\sqrt{5}$

$$p=\frac{2}{\sqrt{5}}\Rightarrow e^2=1+\frac{q^2}{p^2}\Rightarrow 25=1+\frac{5q^2}{4}\Rightarrow q^2=\frac{96}{5}$$

So length of LR $\frac{2q^2}{p}=\frac{96}{\sqrt{5}}$

12. The value of $\int_{-\pi/6}^{\pi/6}\left(\frac{\pi+4x^{11}}{1-\sin(|x|+\pi/6)}\right)dx$ is equal to

(1) 2π (2) 4π

(3) 8π (4) 6π

Ans. (2)

Sol. $=2\pi\int_0^{\pi/6}\frac{1}{1-\sin\left(x+\frac{\pi}{6}\right)}dx$ let $x+\frac{x}{6}=t$ $dx=dt$

$$=2\pi\int_{\pi/6}^{\pi/3}\frac{dt}{1-\sin t}=2\pi\int_{\pi/6}^{\pi/3}\frac{1+\sin t}{\cos^2 t}dt$$

$$=2\pi\left[\int_{\pi/6}^{\pi/3}\sec^2 t\,dt+\int_{\pi/6}^{\pi/3}\sec t\tan t\,dt\right]$$

$$=2\pi\left[(\tan t)_{\pi/6}^{\pi/3}+(\sec t)_{\pi/6}^{\pi/3}\right]$$

$$=2\pi\left[\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)+\left(2-\frac{2}{\sqrt{3}}\right)\right]$$

$$==2\pi\left[\sqrt{3}+2-\sqrt{3}\right]=4\pi$$

13. Let the mean and variance of 7 observations 2, 4, 10, x, 12, 14, y, x > y, be 8 and 16 respectively. Two numbers are chosen from {1, 2, 3, x–4, y, 5} one after another without replacement, then the probability, that the smaller number among the two chosen numbers is less than 4, is:

(1) $\frac{3}{5}$ (2) $\frac{4}{5}$

(3) $\frac{2}{5}$ (4) $\frac{1}{3}$

Ans. (2)

Sol. Mean $(\bar{x})=8$ (Given)

$$\Rightarrow\frac{2+4+10+x+12+14+y}{7}=8$$

$\Rightarrow x+y=14$ ….(1)….

Variance $(\sigma^2)=16$ (Given)

$$\Rightarrow 16=\frac{2^2+4^2+10^2+x^2+12^2+14^2+y^2}{7}-8^2$$

$\Rightarrow x^2+y^2=100$ …..(2)…..

$\because (x+y)^2=x^2+y^2+2xy$

$\Rightarrow xy=48$ (sum is 14 product is 48)

Since problem states $x>y$

$\therefore$ x = 8 and y = 6

Now set X = {1,2,3,4,6,5}

Now we choose two numbers one after author without replacement total outcomes = 6 × 5 = 30

We want the prob. That the smaller number among the two is less than 4

P(smaller < 4) = 1 – P (smaller ≥ 4)

$$=1-\frac{6}{30}=\frac{4}{5}$$

14. Let (α, β, γ) be the co-ordinates of the foot of the perpendicular drawn from the point (5, 4, 2) on the line $\vec{r}=(-\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}-\hat{k})$.

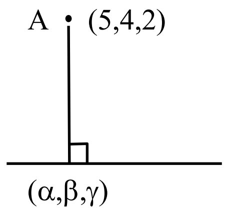
Then the length of the projection of the vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ on the vector $6\hat{i}+2\hat{j}+3\hat{k}$ is :

(1) $\frac{15}{7}$ (2) 4

(3) $\frac{18}{7}$ (4) 3

Ans. (3)

Sol.



$$\bar{r}=\left(-\hat{i}+3\hat{j}+\hat{k}\right)+\lambda(2\hat{i}+3\hat{j}-\hat{k})$$

$$\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}=\lambda$$

Any general point P on the line is

$(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$

Let the given point is A (5,4,2)

$\overline{AP}(2\lambda-6)\hat{i}+(3\lambda-1)\hat{j}+(-\lambda-1)\hat{k}$

$\because\ \overline{AP}\perp^{r}$ Line (L)

$\therefore\ \overline{AP}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)=0$

$2(2\lambda - 6) + 3(3\lambda - 1) - 1(-\lambda - 1) = 0$

$\Rightarrow \lambda = 1$

$\therefore\ \alpha = 1$

$\beta = 6$

$\gamma = 0$

Let the vector $\bar{u}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

$\bar{u}=\hat{i}+6\hat{j}+O\hat{k}$

& $\bar{w}=6\hat{i}+2\hat{j}+3\hat{k}$

So projection $=\frac{|\bar{u}.\bar{w}|}{|\bar{w}|}=\frac{18}{7}$

15. Let PQ and MN be two straight lines touching the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ at the points A and B respectively. Let O be the centre of the circle and ∠AOB = π/3. Then the locus of the point of intersection of the lines PQ and MN is:

(1) $3(x^2 + y^2) - 18x - 12y + 25 = 0$

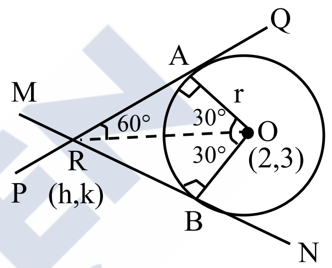
(2) $x^2 + y^2 - 12x - 18y - 25 = 0$

(3) $x^2 + y^2 - 18x - 12y - 25 = 0$

(4) $3(x^2 + y^2) - 12x - 18y - 25 = 0$

Ans. (4)

Sol. Given circle



$x^2 + y^2 - 4x - 6y - 3 = 0$

C(2,3) & r = 4

$$\cos 30° = \frac{r}{OR}=\frac{4}{OR}$$

$$\Rightarrow OR = \frac{8}{\sqrt{3}}$$

Now

$OR^2 = (h - 2)^2 + (k - 3)^2$

$\Rightarrow 3(x^2 + y^2) - 12x - 18y - 25 = 0$

16. If the coefficient of x in the expansion of $(ax^2 + bx + c)(1 - 2x)^{26}$ is – 56 and the coefficients of x^2 and x^3 are both zero, then a + b + c is equal to

(1) 1300 (2) 1500

(3) 1403 (4) 1483

Ans. (3)

Sol. $(ax^2 + bc + c)\sum_{r=0}^{26} {}^{26}C_r(-2x)^r$

Coeff. of x^2 : $a.{}^{26}C_0\ (-\ 2)^0 + b.{}^{26}C_1(-2) + c.{}^{26}C_2(-2)^2=0$

$\Rightarrow a - 52b + 1300c = 0\ \ldots (1)$

Coeff. of x^3 : $a.{}^{26}C_1\ (-\ 2) + b.{}^{26}C_2(-2)^2 + c.{}^{26}C_3(-2)^3=0$

$\Rightarrow -52a + 1300b - 20800c = 0\ \ldots (2)$

Coeff. of $x = -56$

$\Rightarrow b.{}^{26}C_0\,(-2)^0 + c.{}^{26}C_1(-2)^1 = -56$

$b - 52c = -56\ \ldots (3)$

After solving (1), (2) & (3)

$a = 1300,\ b = 100,\ c = 3$

$\Rightarrow a + b + c = 1403$

17. If $x^2 + x + 1 = 0$, then the value of $\left(x+\frac{1}{x}\right)^4+\left(x^2+\frac{1}{x^2}\right)^4+\left(x^3+\frac{1}{x^3}\right)^4+\ldots+\left(x^{25}+\frac{1}{x^{25}}\right)^4$ is :

(1) 128 (2) 162

(3) 175 (4) 145

Ans. **(4)**

Sol. $x^2 + x + 1 = 0$

$\Rightarrow x = \omega$ or ω^2

$\therefore \alpha = \omega\ \&\ \beta = \omega^2$

$= (\omega + \omega^2)^4 + (\omega^2 + \omega^4)^4 + (\omega^3 + \omega^6)^4 + \ldots + (\omega^{25} + \omega^{50})^4$

$= [(\omega + \omega^2)^4 + (\omega^2 + \omega^4)^4 + (\omega^4 + \omega^8)^4 + \ldots + (\omega^{25} + \omega^{50})^4] + [(\omega^3 + \omega^6)^4 + (\omega^6 + \omega^{12})^4 + (\omega^9 + \omega^{18})^4 + \ldots + (\omega^{24} + \omega^{48})^4]$

$= \underbrace{[1+1+1.....+1]}_{17\text{times}}+\underbrace{[(1+1)^4+(1+1)^4+.....(1+1)^4]}_{8\text{times}}$

$= 17 + 128$

$= 145$

18. The value of $\operatorname{cosec}10° - \sqrt{3}\ \sec 10°$ is equal to:

(1) 4 (2) 2

(3) 8 (4) 6

Ans. **(1)**

Sol. $= \dfrac{1}{\sin 10°} - \dfrac{\sqrt{3}}{\cos 10°}$

$= \dfrac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10°\cos 10°}$

$= 4\left[\dfrac{\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°}{2\sin 10°\cos 10°}\right]$

$= 4\left[\dfrac{\sin(30° - 10°)}{\sin 20°}\right]$

$= 4$

19. The sum of all the roots of the equation $(x-1)^2 - 5\,|x-1| + 6 = 0$, is:

(1) 4 (2) 3

(3) 1 (4) 5

Ans. **(1)**

Sol. Let $|x-1| = t$

$t^2 - 5t + 6 = 0$

$t = 2\ \&\ t = 3$

$|x-1| = 2\ \&\ |x-1| = 3$

$x - 1 \pm 2\ \&\ x - 1 = \pm 3$

$x = 1 \pm 2\ \&\ x = 1 \pm 3$

$\therefore$ root $= 3, -1, 4, -2$

$\therefore$ Sum of root $= 3 + (-1) + 4 + (-2) = 4.$

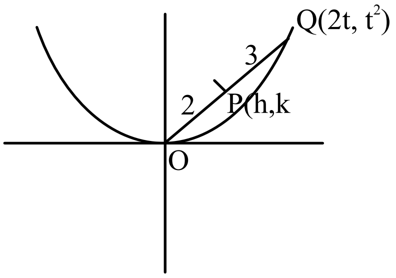
20. Let O be the vertex of the parabola $x^2 = 4y$ and Q be any point on it. Let the locus of the point P, which divides the line segment OQ internally in the ratio 2: 3 be the conic C. Then the equation of the chord of C, which is bisected at the point (1, 2), is:

(1) $5x-y-3=0$ (2) $4x-5y+6=0$

(4) $x-2y+3=0$ (4) $5x-4y+3=0$

Ans. **(4)**

Sol. $h = \dfrac{4t}{5}$



$$k = \frac{2t^2}{5} = \frac{2}{5}\left(\frac{5h}{4}\right)^2$$

$8k = 5h^2$

$\Rightarrow 5x^2 = 8y$

$T = S_1$

$5(xx_1) - 4(y + y_1) = 5x_1^2 - 8y_1$

$5(x) - 4(y + 2) = 5 - 8.2$

$5x - 4y + 3 = 0$

SECTION-B

21. Let $f : R \to R$ be a twice differentiable function such that the quadratic equation $f(x)m^2 - 2f'(x)m + f''(x) = 0$ in m, has two equal roots for every $x \in R$. If $f(0)=1$, $f'(0) = 2$ and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha + \beta$ is equal to

Ans. **(1)**

Sol. Given quadratic equation has equal roots, thus

$D = 0 \Rightarrow (f'(x))^2 = f''(x) \cdot f(x)$

$$\frac{f'(x)}{f(x)} = \frac{f''(x)}{f'(x)}$$

Integrate

$\ell n\,(f(x)) = \ell n\,(f'(x)) + \ell nC \Rightarrow f(x) = c.f'(x)$

Put $= x = 0$

$1 = c.2 \Rightarrow c = \dfrac{1}{2}$

Now $2f(x) = f'(x)$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2$$

Integrate

$\ell n\,(f(x)) = 2x + d$

$\Rightarrow d = 0$

$\Rightarrow \ell n\,(f(x)) = 2x \Rightarrow f(x) = e^{2x}$

Now let $g(x) = f(\ell n\, x - x) = e^{2(\ell nx - x)}$

$$\therefore g'(x) = 2\,e^{2(\ell n x - x)}\left(\frac{1}{x} - 1\right) \geq 3$$

$$\Rightarrow \frac{1-x}{x} \geq 0$$

$\Rightarrow x \in (0,1]$

$\Rightarrow \alpha = 0, \beta = 1$

$\alpha + \beta = 1.$

22. Let $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = \dfrac{1}{2}a_n + \dfrac{n^2 - 2n - 1}{n^2(n+1)^2}$. Then $\left|\displaystyle\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right|$ is equal to ________.

Ans. **(2)**

Sol. $$a_{n+1} - \frac{1}{2}a_n = \frac{n^2 - 2n - 1}{n^2(n+1)^2} = \frac{2n^2 - (n+1)^3}{n^2(n+1)^2}$$

$$\Rightarrow a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2}$$

$$n = 1 \quad a_2 - \frac{1}{2}a_1 = \frac{2}{2^2} - \frac{1}{1^2}$$

$$2\left[a_3 - \frac{1}{2}a_2 = \frac{2}{3^2} - \frac{1}{2^2}\right]$$

$$2^2\left[a_4 - \frac{1}{2}a_3 = \frac{2}{4^2} - \frac{1}{3^2}\right]$$

$$\vdots$$

$$2^{n-2}\left[a_n - \frac{1}{2}a_{n-1} = \frac{2}{n^2} - \frac{1}{(n-1)^2}\right]$$

$$2^{n-1}\left[a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2}\right]$$

Adding

$$a_{n+1} = \frac{2}{(n+1)^2} - \frac{1}{2^n} \Rightarrow a_n = \frac{2}{n^2} - \frac{1}{2^{n-1}}$$

$$\Rightarrow \left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right| \Rightarrow \left|\sum_{n=1}^{\infty} -\frac{1}{2^{n-1}}\right| = 2$$

23. Let S={(m, n): m, n ∈ {1, 2, 3, ..., 50}}. If the number of elements (m, n) in S such that $6^m + 9^n$ is a multiple of 5 is p and the number of elements (m, n) in S such that m + n is a square of a prime number is q, then p + q is equal to……..

Ans. **(1333)**

Sol. S = {1, 2, 3,….50}

p = $(6^m + 9^n)$ is divisible by 5

No. of ways

$6^m = (5\lambda + 1)^m = 5k + 1$

$9^n = (10 - 1)^n = 10\mu - 1$ if n is odd

$\Rightarrow$ n must be odd

$10\mu + 1$ if n is even

$\Rightarrow$ No. of ways = 50 × 25 = 1250

q $\Rightarrow$ (m + n) is square of a prime

	m + n = 4	m + n = 9	m + n = 25	m + n = 49
No.of	↓	↓	↓	↓
ways	3	8	24	48

q = 3 + 8 + 24 + 48 = 83

= p + q = 1250 + 83 = 1333

24. For some α, β ∈ R, let A = $\begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ be such that $A^2 - 4A + 2I = B^2 - 3B + I = O$. Then $(\det(\text{adj}(A^3 - B^3)))^2$ is equal to ………

Ans. **(225)**

Sol. Tr (A) = 4 $\Rightarrow \alpha + 2 = 4 \Rightarrow \alpha = 2$

Tr (B) = 3 $\Rightarrow \beta + 1 = 3 \Rightarrow \beta = 2$

$A^2 - 4A + 2I = O$

$A^3 = 4A^2 - 2A = 16A - 8I - 2A$

14A – 8I

$$= \begin{bmatrix} 28 & 28 \\ 14 & 28 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix}$$

$$= A^3 = \begin{bmatrix} 20 & 28 \\ 14 & 20 \end{bmatrix}$$

$B^2 - 3B + I = O$

$B^2 = 3B - I$

$\Rightarrow B^3 = 3B^2 - B = 3(3B - I) - B = 8B - 3I$

$$B^3 = \begin{pmatrix} 8 & 8 \\ 8 & 16 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$$

$$A^3 - B^3 = \begin{pmatrix} 20 & 28 \\ 14 & 20 \end{pmatrix} - \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix} = \begin{pmatrix} 15 & 20 \\ 6 & 7 \end{pmatrix}$$

$\Rightarrow |A^3 - B^3| = 105 - 120 = -15$

$\Rightarrow |\text{adj}(A^3 - B^3)| = |A^3 - B^3| = -15$

$\Rightarrow |\text{adj}(A^3 - B^3)|^2 = 225$

25. $6\int_0^{\pi} |(\sin 3x + \sin 2x + \sin x)|\,dx$ is equal to……..

Ans. **(17)**

Sol. $6\int\limits_0^{\pi} |2\sin 2x \cos x + \sin 2x|\,dx$

$6\int\limits_0^{\pi} |4\sin x \cos^2 x + 2\sin x \cos x|\,dx$

$I = 12\int\limits_0^{\pi} \sin x\,|2\cos^2 x + \cos x|$

Put cosx = t, –sinx dx = dt

$I = -12\int\limits_1^{-1} |2t^2 + t|\,dt$

I =

$$12\left(\int\limits_{-1}^{-1/2} (2t^2 + t)dt + \int\limits_{-1/2}^{0} -(2t^2 + t)dt + \int\limits_0^1 (2t^2 + t)dt\right)$$

I = 17