JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON THURSDAY 22nd JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 5k$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + \lambda k, \lambda \in \mathbb{R}$. Let the projection of the vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ on the diagonal $\overrightarrow{AC}$ of the parallelogram ABCD be of length one unit. If α, β, where α > β, be the roots of the equation $\lambda^2x^2 - 6\lambda x + 5 = 0$, then 2α – β is equal to

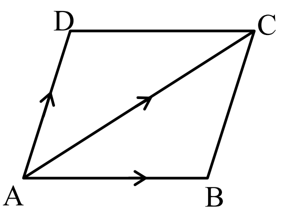
(1) 1 (2) 4

(3) 3 (4) 6

Ans. (3)

Sol. $\overrightarrow{AC} = 3\hat{i} + 6\hat{j} + (\lambda - 5)k$

$\vec{v} \cdot AC = 1 \Rightarrow 3 + 6 + \lambda - 5 = \sqrt{9 + 36 + (\lambda - 5)^2}$



$\Rightarrow \lambda^2 + 8\lambda + 16 = \lambda^2 - 10\lambda + 70$

$\Rightarrow \lambda = \dfrac{54}{18} = 3$

$\therefore$ Quadratic : $9x^2 - 18x + 5 = 0 \Rightarrow x = \dfrac{1}{3}, \dfrac{5}{3}$

$\therefore 2\alpha - \beta = \dfrac{10 - 1}{3} = 3$

2. Let the relation R on the set M = {1, 2, 3,.......16} be given by

R = {(x,y) : 4y = 5x – 3, x, y ∈ M}.

Then the minimum number of elements required to be added in R, in order to make the relation symmetric, is equal to

(1) 1 (2) 2

(3) 4 (4) 3

Ans. (2)

Sol. R = {(3, 3), (7, 8), (11, 13)}

to make it symmetric (8, 7), (13, 11) must be added.

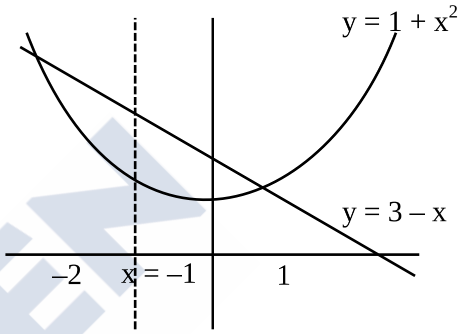
3. Let the line x = –1 divide the area of the region $\{(x, y) : 1 + x^2 \le y \le 3 - x\}$ in the ratio m : n, gcd (m, n) = 1. Then m + n is equal to

(1) 25 (2) 28

(3) 26 (4) 27

Ans. (4)

Sol.



$$\frac{m}{n} = \frac{\int_{-1}^{1}\left[(3 - x) - (1 + x^2)\right]dx}{\int_{-2}^{1}\left[(3 - x) - (1 + x^2)\right]dx} = \frac{20}{7}$$

$\therefore$ m + n = 20 + 7

= 27

4. Two distinct numbers a and b are selected at random from 1, 2, 3,......, 50. The probability, that their product ab is divisible by 3, is

(1) $\dfrac{561}{1225}$ (2) $\dfrac{664}{1225}$

(3) $\dfrac{272}{1225}$ (4) $\dfrac{8}{25}$

Ans. (2)

Sol. Req. probability = 1– (product not divisible by 3)

Multiple of 3 = 16

Not multiple of 3 = 34

$= 1 - \dfrac{^{34}C_2}{^{50}C_2} = \dfrac{664}{1225}$

5. Let $f(x) = x^{2025} - x^{2000}$, $x \in [0, 1]$ and the minimum value of the function f(x) in the interval [0, 1] be $(80)^{80}(n)^{-81}$. Then n is equal to

(1) –81 (2) –40

(3) –41 (4) –80

Ans. **(1)**

Sol. $f(x) = x^{2025} - x^{2000}$

$$f'(x) = 0 \Rightarrow x = \left(\frac{2000}{2025}\right)^{1/25} = \alpha(\text{say})$$

$$\therefore f(0) = 0, f(1) = 0, f(\alpha) = \left(\frac{80}{81}\right)^{80} \cdot \frac{-1}{81} = 80^{80}.(-81)^{-81}$$

6. Let $P(\alpha, \beta, \gamma)$ be the point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point (1, –1, 0) and nearer to the origin. Then the shortest distance, between the lines $\frac{x-\alpha}{1} = \frac{y-\beta}{2} = \frac{z-\gamma}{3}$ and $\frac{x+5}{2} = \frac{y-10}{1} = \frac{z-3}{1}$, is equal to

(1) $7\sqrt{\frac{5}{4}}$ (2) $4\sqrt{\frac{7}{5}}$

(3) $4\sqrt{\frac{5}{7}}$ (4) $2\sqrt{\frac{7}{4}}$

Ans. **(2)**

Sol. Let $P(2\lambda+1, -3\lambda-1, \lambda)$

Then $4\lambda^2 + 9\lambda^2 + \lambda^2 = 16 \cdot 14 \Rightarrow \lambda = \pm 4 \Rightarrow -4$ (nearer to origin)

$\therefore$ P(–7, 11, –4)

$$\therefore \text{ Shortest distance} = \frac{\begin{vmatrix} 2 & -1 & 7 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}}{\left|\begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}\right|}$$

$$= \frac{28}{\sqrt{1+25+9}} = \frac{4\sqrt{7}}{\sqrt{5}}$$

7. If a random variable x has the probability distribution

x	0	1	2	3	4	5	6	7
p(x)	0	2k	k	3k	$2k^2$	2k	k^2+k	$7k^2$

then $P(3 < x \le 6)$ is equal to

(1) 0.34 (2) 0.22

(3) 0.64 (4) 0.33

Ans. **(4)**

Sol. $\sum P(x_i) = 1$

$\Rightarrow 9k + 10k^2 = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow k = \frac{1}{10}$

$P(3 < x \le 6) = 3k + 3k^2$

$= \frac{3}{10} + \frac{3}{100} = 0.33$

$= 0.33$

8. The number of distinct real solutions of the equation $x|x + 4| + 3|x + 2| + 10 = 0$ is

(1) 3 (2) 1

(3) 0 (4) 2

Ans. **(2)**

Sol. Case I $x < -4$

$x(-(x+4)) + 3(-(x+2)) + 10 = 0$

$x^2 + 7x - 4 = 0$

$\Rightarrow x = -\frac{7+\sqrt{65}}{2}$ or $-\frac{7-\sqrt{65}}{2}$

Reject Acept

Case II $-4 \le x < -2$

$x(x + 4) + 3(-(x+2)) + 10 = 0$

$x^2 + x + 4 = 0$

$D < 0$ No solution

Case III $x \ge -1$

$x(x + 4) + 3(x + 2) + 10 = 0$

$x^2 + 7x + 16 = 0$

$D < 0$ No solution

$\Rightarrow$ No. of solution = 1.

9. Let $f : [1, \infty) \to \mathbb{R}$ be a differentiable function, If $6\int_1^x f(t)dt = 3xf(x) + x^3 - 4$ for all $x \geq 1$, then the value of $f(2) - f(3)$ is

(1) –4 (2) –3

(3) 4 (4) 3

Ans. (4)

Sol. $6\int_1^x f(t)dt = 3x\ f(x) + x^3 - 4$

Diff. both side

$6\ f(x) = 3x\ f'(x) + 3f(x) + 3x^2$

$3f(x) = 3x\ f'(x) + 3x^2$

$x\dfrac{dy}{dx} - y = -x^2$

$\dfrac{x\dfrac{dy}{dx} - 9}{x^2} = -1$

$\Rightarrow \dfrac{d}{dx}\left(\dfrac{y}{x}\right) = -1$

$\dfrac{y}{x} = -x + C$

$\Rightarrow f(x) = -x^2 + Cx$

at $x = 1$, $y = 1 \Rightarrow C = 2$

$f(x) = -x^2 + 2x$

$f(2) - f(3) = 3$

10. If the line $\alpha x + 2y = 1$, where $\alpha \in \mathbb{R}$, does not meet the hyperbola $x^2 - 9y^2 = 9$, then a possible value of α is :

(1) 0.6 (2) 0.8

(3) 0.5 (4) 0.7

Ans. (2)

Sol. $y = \dfrac{1-\alpha x}{2}$

Put this in equation of hyperbola

$\therefore x^2 - 9\left(\dfrac{1-\alpha x}{2}\right)^2 = 9$

$(4-9\alpha^2)x^2 + 18\alpha x - 45 = 0$

$\because$ line does not intersect hyperbola

$\therefore D < 0$

$\Rightarrow \alpha^2 - \dfrac{5}{9} > 0$

$\Rightarrow \alpha \in \left(-\infty, -\dfrac{\sqrt{5}}{3}\right) \cup \left(\dfrac{\sqrt{5}}{3}, \infty\right)$

Here $\dfrac{\sqrt{5}}{3} \simeq 0.74$

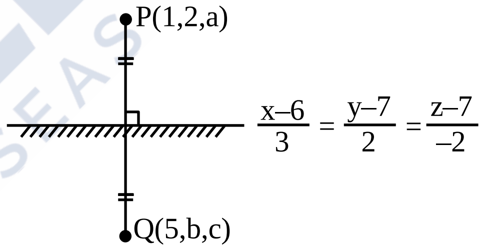
11. If the image of the point P (1, 2 , a) in the line $\dfrac{x-6}{3} = \dfrac{y-7}{2} = \dfrac{7-z}{2}$ is Q(5, b, c), then $a^2 + b^2 + c^2$ is equal to

(1) 293 (2) 264

(3) 298 (4) 283

Ans. (3)

Sol.



Point $M \equiv \left(3, \dfrac{b}{2}+1, \dfrac{c+a}{2}\right)$ satisfies the line

$\dfrac{3-6}{3} = \dfrac{\dfrac{b}{2}+1-7}{2} = \dfrac{\dfrac{c+a}{2}-7}{-2}$

$-1 = \dfrac{b-12}{4} = \dfrac{c+a-14}{-4}$

$\Rightarrow b = 8 \ \ldots (1)$ & $c + a = 18 \ldots (2)$

Now $PQ \perp L$

$\Rightarrow (4i + (b-2)j + (c-a)k) \cdot (3i + 2j - 2k) = 0$

$\Rightarrow 12 + 2(b-2) - 2(c-a) = 0$

$\Rightarrow 6 + (b-2) - (c-a) = 0$

$\Rightarrow b - c + a + 4 = 0$

$\Rightarrow 8 - c + a + 4 = 0$

$\Rightarrow c + a = 12 \ \ldots (3)$

From (2) & (3)

$c = 15$ & $a = 3$

So $a^2 + b^2 + c^2 = 9 + 64 + 225 = 298$

12. Let the set of all values of r, for which the circles $(x + 1)^2 + (y + 4)^2 = r^2$ and $x^2 + y^2 - 4x - 2y - 4 = 0$ intersect at two distinct points be the interval (α, β). Then $\alpha\beta$ is equal to

(1) 25 (2) 20

(3) 21 (4) 24

Ans. **(1)**

Sol. $(x-2)^2 + (y-1)^2 = 3^2$ & $(x+1)^2 + (y+4)^2 = r^2$

$|r_1 - r_2| < c_1c_2 < r_1 + r_2$

$$|r-3| < \sqrt{(2+1)^2 + (1+4)^2} < r+3$$

$$|r-3| < \sqrt{34} \ \ \& \ r+3 > \sqrt{34}$$

$$-\sqrt{34} < r-3 < \sqrt{34} \ \ \& \ \ r > \sqrt{34} - 3$$

$$\text{i.e. } r = \left(3-\sqrt{34}, 3+\sqrt{34}\right) \cap \left(\sqrt{34}-3, \infty\right)$$

$$\text{i.e. } r \in \left(\sqrt{34}-3, \sqrt{34}+3\right)$$

$$\therefore \alpha\beta = \left(\sqrt{34}-3\right)\left(\sqrt{34}+3\right)$$

$= 34 - 9$

$= 25$

13. If $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$, then the determinant of the matrix $(A^{2025} - 3A^{2024} + A^{2023})$ is

(1) 28 (2) 12

(3) 24 (4) 16

Ans. **(4)**

Sol. $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 13 & 21 \\ 21 & 34 \end{bmatrix}$

$|A^{2025} - 3A^{2024} + A^{2023}|$

$$= \left| A^{2023}\left(A^2 - 3A + I\right) \right|$$

$$= |A|^{2023} \left| A^2 - 3A + I \right|$$

$$= 1.\begin{vmatrix} 8 & 12 \\ 12 & 20 \end{vmatrix} = 160 - 144 = 16$$

14. If the domain of the function $f(x) = \sin^{-1}\left(\frac{5-x}{3+2x}\right) + \frac{1}{\log_e(10-x)}$ is $(-\infty, \alpha] \cup [\beta, \gamma) - \{\delta\}$, then $6(\alpha + \beta + \gamma + \delta)$ is equal to

(1) 70 (2) 66

(3) 67 (4) 68

Ans. **(1)**

Sol. $-1 \le \frac{5-x}{2x+3} \le$ & $10 - x > 0$, $10 - x \ne 1$

$$\left|\frac{5-x}{2x+3}\right| \ge 1 \ \ \& \ x < 10 \ \ \& \ x \ne 9$$

$(5 - x)^2 - (2x + 3)^2 \le 0$ & $x < 10$ & $4x \ne 9$

$(x + 8)(3x - 2) \ge 0$ & $x < 10$ & $x \ne 9$

$$\Rightarrow \left(-\infty, -8\right] \cup \left[\frac{2}{3}, 10\right) - \{9\}$$

$$\Rightarrow (\alpha + \beta + \gamma + \delta) = 6\left(-8 + \frac{2}{3} + 10 + 9\right)$$

$= 70$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{[x]+4}\right)dx$, where $[\bullet]$ denotes the greatest integer function, is

(1) $\frac{1}{60}(21\pi - 1)$ (2) $\frac{1}{60}(\pi - 7)$

(3) $\frac{7}{60}(3\pi - 1)$ (4) $\frac{7}{60}(\pi - 3)$

Ans. **(3)**

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{[x]+4}dx$

$$I = \int_{-\pi/2}^{-1} \frac{dx}{2} + \int_{-1}^{0} \frac{dx}{3} + \int_{1}^{0} \frac{dx}{4} + \int_{1}^{\pi/2} \frac{dx}{5}$$

$$= \frac{1}{2}\left(-1 + \frac{\pi}{2}\right) + \frac{1}{3}(1) + \frac{1}{4}(1) + \left(\frac{\pi}{2} - 1\right)\frac{1}{5}$$

$$= \frac{7\pi}{20} - \frac{7}{60} = \frac{7}{60}(3\pi - 1)$$

16. The coefficient of x^{48} in $(1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + ... + 100(1 + x)^{100}$ is equal to :

(1) $100.\,{}^{100}C_{49} - {}^{100}C_{50}$ (2) ${}^{100}C_{50} + {}^{101}C_{49}$

(3) $100.\,{}^{100}C_{49} - {}^{100}C_{48}$ (4) $100.\,{}^{101}C_{49} - {}^{100}C_{50}$

Ans. (4)

Sol. Let $1 + x = r$

$\therefore S = 1.r + 2.r^2 + 3.r^3 + \ldots\ldots + 100r^{100}$ .... (1)

(AGP)

$rS = 1.r^2 + 2.r^3 + \ldots\ldots + 99r^{100} + 100r^{101}$ .... (2)

(1) – (2) gives

$$S = -\frac{(1+x)^{101}}{x^2} + \frac{1}{x^2} + \frac{100(1+x)^{101}}{x}$$

$\therefore$ coefficient x^{48} in S

$= -$ coefficient of x^{48} in $\dfrac{(1+x)^{101}}{x^2} + 100$.

Coefficient of x^{48} in $\dfrac{(1+x)^{101}}{x}$

$= 100\,{}^{101}C_{49} - {}^{101}C_{50}$

17. If the chord joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the parabola $y^2 = 12x$ subtends a right angle at the vertex of the parabola, then $x_1x_2 - y_1y_2$ is equal to

(1) 288 (2) 280

(3) 284 (4) 292

Ans. (1)

Sol. $(x_1y_1) = (3t_1^2, 6t_1)$ & $(x_2y_2) = (3t_2^2, 6t_2)$

$t_1t_2 = -4$

$x_1x_2 = 9(t_1t_2)^2,\ y_1y_2 = 36t_1t_2$

$x_1x_2 - y_1y_2 = 9(16) - 36(-4)$

$= 144 + 144$

$= 288$

18. The number of solutions of $\tan^{-1}4x + \tan^{-1}6x = \dfrac{\pi}{6}$, where $-\dfrac{1}{2\sqrt{6}} < x < \dfrac{1}{2\sqrt{6}}$ is equal to

(1) 3 (2) 0

(3) 1 (4) 2

Ans. (3)

Sol. $\tan^{-1}4x + \tan^{-1}6x = \dfrac{\pi}{6}$

$$\Rightarrow \tan^{-1}\left(\frac{4x+6x}{1+24x^2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{10x}{1-24x^2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 24x^2 + 10\sqrt{3}x - 1 = 0$$

$$x = \frac{-10\sqrt{3} \pm \sqrt{300+96}}{48}$$

$$x = \frac{\sqrt{396} - 10\sqrt{3}}{48}$$

Only 1 solution in $\left(-\dfrac{1}{2\sqrt{6}}, \dfrac{1}{2\sqrt{6}}\right)$

19. Let the solution curve of the differential equation $xdy - ydx = \sqrt{x^2 + y^2}\,dx$, $x > 0$, $y(1) = 0$, be $y = y(x)$. Then $y(3)$ is equal to

(1) 4 (2) 6

(3) 1 (4) 2

Ans. (1)

Sol. $\dfrac{xdy - ydx}{x^2} = \dfrac{\sqrt{x^2+y^2}}{x^2}dx$

$$d\left(\frac{y}{x}\right) = \sqrt{1+\frac{y^2}{x^2}}.\frac{1}{x}dx$$

$$\int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \int \frac{1}{x}dx$$

$$\Rightarrow \ln\left(\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}\right) = \ln x + \ln k = \ln kx$$

$$\Rightarrow y + \sqrt{x^2+y^2} = kx^2$$

$0 + 1 = k$

$$\Rightarrow y + \sqrt{x^2+y^2} = x^2$$

$$y + \sqrt{9+y^2} = 9$$

$y = 4$

20. If the sum of the first four terms of an A.P. is 6 and the sum of its first six terms is 4, then the sum of its first twelve terms is

(1) –20 (2) –24

(3) –26 (4) –22

Ans. (4)

Sol. Sum of first 4 term $S_4 = 6$

$$\frac{4}{2}(2a+3d)=6 \Rightarrow 2a+3d=3 \quad \text{..... (1)}$$

Sum of first 6 terms $S_6 = 4$

$$\frac{6}{2}(2a+5d)=4 \Rightarrow 2a+5d=\frac{4}{3} \quad \text{..... (2)}$$

eq. (2) – eq. (1)

$$(2a+5d)-(2a+3d)=\frac{4}{3}-3$$

$$\Rightarrow d=-\frac{5}{6}$$

$$\therefore\ 2a+3\left(-\frac{5}{6}\right)=3 \Rightarrow a=\frac{11}{4}$$

$$S_{12}=\frac{12}{2}\left\{2\times\frac{11}{4}+(12-1)\left(-\frac{5}{6}\right)\right\}$$

$$S_{12}=6\left(-\frac{22}{6}\right)=-22$$

SECTION-B

21. Let $\alpha=\frac{-1+i\sqrt{3}}{2}$ and $\beta=\frac{-1-i\sqrt{3}}{2}$, $i=\sqrt{-1}$. If $(7-7\alpha+9\beta)^{20}+(9+7\alpha-7\beta)^{20}+(-7+9\alpha+7\beta)^{20}+(14+7\alpha+7\beta)^{20}=m^{10}$, then m is ________.

Ans. (49)

Sol. $\left(9+7\omega-7\omega^2\right)+\omega^{20}\left(9+7\omega-7\omega^2\right)^{20}+$

$\omega^{40}\left(9+7\omega-7\omega^2\right)^{20}+(14+7(\omega+\omega^2))^{20}$

$\left(9+7\omega-7\omega^2\right)^{20}\left(1+\omega+\omega^2\right)+(14-7)^{20}=7^{20}$

$=(49)^{10}$

Hence, M = 49

22. Let A be a 3 × 3 matrix such that $A + A^T = O$. If $A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}3\\3\\2\end{bmatrix}, A^2\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}-3\\19\\-24\end{bmatrix}$ and det(adj(2adj(A + I))) = $(2)^{\alpha}.(3)^{\beta}.(11)^{\gamma}$, α, β, γ are non-negative integers, then $\alpha+\beta+\gamma$ is equal to ______

Ans. (18)

Sol. $A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}3\\3\\2\end{bmatrix}$ and $A\begin{bmatrix}3\\3\\2\end{bmatrix}=\begin{bmatrix}-3\\19\\-24\end{bmatrix}$

det. (adj.(2 adj (A+I)) = $|2 \text{ adj } (A+I)|^2 = 64\ |\text{adj } (A+I)|^2$

$= 64\ |A+I|^4$

Let $A=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}\Rightarrow\left\{\begin{matrix}-a=3\\-b+c=2\\3a+2b=-3\end{matrix}\right\}$

$\Rightarrow\left\{\begin{matrix}a=-3\\b=3\\c=5\end{matrix}\right.$

$\therefore\ |A+I| = \begin{vmatrix}1&-3&3\\3&1&5\\-3&-5&1\end{vmatrix}=44$

23. If $\int(\sin x)^{\frac{-11}{2}}(\cos x)^{\frac{-5}{2}}dx=$

$-\frac{p_1}{q_1}(\cot x)^{\frac{9}{2}}-\frac{p_2}{q_2}(\cot x)^{\frac{5}{2}}-\frac{p_3}{q_3}(\cot x)^{\frac{1}{2}}+\frac{p_4}{q_4}(\cot x)^{\frac{-3}{2}}+C$,

where p_i and q_i are positive integers with gcd(p_i, q_i) = 1 for i = 1, 2, 3, 4 and C is the constant of integration, then $\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}$ is equal to _______.

Ans. (16)

Sol. $\int (\tan x)^{-11/2} .\sec^8 x\, dx$

$= \int (\tan x)^{-11/2} \left(1+\tan^2 x\right) 3\sec^2 x dx$

Put tan x = t

$\Rightarrow \int t^{-11/2}\left(1+t^2\right)^3 dx = \int t^{-11/2}\left(1+t^6+3t^2+3(4)\right)dt$

$= \int \left(t^{-11/2}+t^{1/2}+3.t^{-7/2}+3.t^{-3/2}\right)dt$

$= -\frac{2}{9}(\cot x)^{9/2} - \frac{6}{5}(\cot)^{5/2} - 6(\cot x)^{1/2} + \frac{2}{3}(\cot x)^{-3/2} + C$

$\Rightarrow p_1 = 2, p_2 = 6, p_3 = 6, p_4 = 2$

& $q_1 = 9, q_2 = 5, q_3 = 1, q_4 = 3$

$\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4} = \frac{15.2.6.6.2}{9.5.1.3} = 16$

24. If $\frac{\cos^2 48° - \sin^2 12°}{\sin^2 24° - \sin^2 6°} = \frac{\alpha+\beta\sqrt{5}}{2}$, where $\alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to ________.

Ans. **(4)**

Sol. Use $\sin(A+B)\sin(A+B) = \sin^2A - \sin^2B$

$\cos(A+B)\cos(A-B) = \cos^2A - \sin^2B$

$\frac{\cos 60° \cos 36°}{\sin 30° \sin 18°} = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2}{4}$

$= \frac{3+\sqrt{5}}{2}$

$\alpha = 3 ; \beta = 1$

So, $(\alpha+\beta) = 4$

25. Let ABC be a triangle. Consider four points p_1, p_2, p_3, p_4 on the side AB, five points p_5, p_6, p_7, p_8, p_9 on the side BC and four points p_{10}, p_{11}, p_{12}, p_{13} on the side AC. None of these points is a vertex of the triangle ABC. Then the total number of pentagons, that can be formed by taking all the vertices from the points p_1, p_2, ... p_{13}, is ________.

Ans. **(660)**

Sol. **Case 1 :**

2 from AB, 2 from BC, 1 from AC

$\binom{4}{2}.\binom{5}{2}.\binom{4}{1} = 6 \cdot 10 \cdot 4 = 240$

Case 2 :

2 from AB, 1 from BC, 2 from AC

$\binom{4}{2}.\binom{5}{1}.\binom{4}{2} = 6 \cdot 5 \cdot 6 = 180$

Case 3 :

1 from AB, 2 from BC, 2 from AC

$\binom{4}{1}.\binom{5}{2}.\binom{4}{2} = 4 \cdot 10 \cdot 6 = 240$