JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON FRIDAY 23rd JANUARY 2026) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTIONS

SECTION-A

1. If $f(x)=\begin{cases}\dfrac{a|x|+x^2-2(\sin|x|)(\cos|x|)}{x} & , x\neq 0\\ b & , x=0\end{cases}$

is continuous at x = 0, then a + b is equal to :

(1) 1 (2) 2

(3) 0 (4) 4

Ans. (2)

Sol. $f(x)=\begin{cases}\dfrac{a|x|+x^2-2\sin|x|\cos|x|}{x} & ; \quad x\neq 0\\ b & ; \quad x=0\end{cases}$

for continuity

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)=f(0)$

$\lim\limits_{x\to 0^-}\dfrac{ah+h^2-2(\sinh)\cosh}{-h}$

$=\lim\limits_{x\to 0^+}\dfrac{ah+h^2-2(\sinh)\cosh}{h}$

or $-a+2=a-2=b$

$2a=4$

$a=2,\ b=0$

$\therefore\ a+b=2$

2. Let $\dfrac{\pi}{2}<\theta<\pi$ and $\cot\theta=-\dfrac{1}{2\sqrt{2}}$. Then the value of

$\sin\left(\dfrac{15\theta}{2}\right)(\cos 8\theta+\sin 8\theta)+\cos\left(\dfrac{15\theta}{2}\right)(\cos 8-\sin 8\theta)$

is equal to :

(1) $\dfrac{1-\sqrt{2}}{\sqrt{3}}$ (2) $-\dfrac{\sqrt{2}}{\sqrt{3}}$

(3) $\dfrac{\sqrt{2}-1}{\sqrt{3}}$ (4) $\dfrac{\sqrt{2}}{\sqrt{3}}$

Ans. (1)

Sol. $\dfrac{\pi}{2}<\theta<\pi$ and $\cot\theta=-\dfrac{1}{2\sqrt{2}}$

$\Rightarrow \sin\left(\dfrac{15\theta}{2}\right)(\cos 8\theta+\sin 8\theta)$

$+\cos\left(\dfrac{15\theta}{2}\right)(\cos 8\theta-\sin 8\theta)$

$\Rightarrow \sin\left(\dfrac{15\theta}{2}\right)\cos 8\theta-\cos\left(\dfrac{15\theta}{2}\right)\sin 8\theta$

$+\sin\dfrac{15\theta}{2}\sin 8\theta+\cos\dfrac{15\theta}{2}\cos 8\theta$

$\Rightarrow \sin\left(\dfrac{15\theta}{2}-8\theta\right)+\cos\left(\dfrac{15\theta}{2}-8\theta\right)$

$\Rightarrow \cos\dfrac{\theta}{2}-\sin\dfrac{\theta}{2}=-\sqrt{1-\sin\theta}\quad\left(\dfrac{\pi}{4}<\dfrac{\theta}{2}<\dfrac{\pi}{2}\right)$

$\Rightarrow$ given $\cot\theta=-\dfrac{1}{2\sqrt{2}}$, $\sin\theta=\dfrac{2\sqrt{2}}{3}$

$\Rightarrow -\sqrt{1-\sin\theta}=\sqrt{\dfrac{3-2\sqrt{2}}{3}}=-\dfrac{(\sqrt{2}-1)}{3}$

$=\dfrac{1-\sqrt{2}}{\sqrt{3}}$

3. Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}+\hat{j}-\hat{k},\ \vec{c}=\lambda\hat{i}+\hat{j}+\hat{k}$ and $\vec{v}=\vec{a}\times\vec{b}$. If $\vec{v}.\vec{c}=11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p, then $9p^2$ is equal to :

(1) 9 (2) 6

(3) 4 (4) 12

Ans. (4)

Sol. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$, and

$\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v}.\vec{c} = 11$

$\vec{v} = (\vec{a} \times \vec{b}) = (-\hat{i} + 7\hat{j} + 5\hat{k})$

$\vec{v}.\vec{c} = 11 = (-\hat{i} + 7\hat{j} + 5\hat{k}).(\lambda\hat{i} + \hat{j} + \hat{k}) = 11$

$\Rightarrow -\lambda + 7 + 5 = 11$

$\Rightarrow \lambda = 1$

Length of projection of $\vec{b}$ on $\vec{c}$ = $\vec{b}.\hat{c}$

$\Rightarrow \left|(2\hat{i} + \hat{j} - \hat{k}).\dfrac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}\right| = \dfrac{2 + 1 - 1}{\sqrt{3}} = P = \dfrac{2}{\sqrt{3}}$

$\Rightarrow 9p^2 = 9\left(\dfrac{4}{3}\right) = 12$

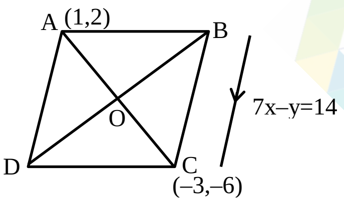
4. Let A(1, 2) and C(–3, –6) be two diagonally opposite vertices of a rhombus, whose sides AD an BC are parallel to the line 7x – y = 14. If B (α, β) and D(γ, δ) are the other two vertices, then $|\alpha + \beta + \gamma + \delta|$ is equal to :

(1) 9 (2) 3

(3) 6 (4) 1

Ans. **(3)**

Sol.



Given the points of B and D are (α, β) and (γ, δ) and mid point of A and C is (–1, –2)

So $\dfrac{\alpha + \gamma}{2} = -1$ and $\dfrac{\beta + \delta}{2} = -2$

$|\alpha + \gamma + \beta + \delta| = 6$

5. Consider two sets $A = \{x \in z : |(|x - 3| - 3)| \le 1\}$ and $B = \left\{x \in \mathbb{R} - \{1,2\} : \dfrac{(x-2)(x-4)}{x-1}\log_e(|x-2|) = 0\right\}$.

Then the number of onto functions f : A→B is equal to :

(1) 62 (2) 79

(3) 32 (4) 81

Ans. **(1)**

Sol. A : $||x - 3| - 3| \le 1$

$\Rightarrow -1 \le |x - 3| - 3 \le 1$

$2 \le |x - 3| \le 4$

$2 \le (x - 3) \le 4$ or $-4 \le (x - 3) \le -2$

$5 \le x \le 7$ or $-1 \le x \le 1$

A = {–1, 0, 1, 5, 6, 7}

B $\Rightarrow$ x = 4, |x – 2| = 1 $\Rightarrow$ x = 3 or 1(reject) +

B = {3, 4}

Number of onto functions from A to B $= 2^6 - 2 = 62$

6. The system of linear equations

x + y + z = 6

2x + 5y + az = 36

x + 2y + 3z = b

has

(1) unique solution for a = 8 and b = 16

(2) infinitely many solutions for a = 8 and b = 14

(3) infinitely many solutions for a = 8 and b = 16

(4) unique solution for a = 8 and b = 14

Ans. **(2)**

Sol. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow a = 8$

If $D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 36 & 5 & a \\ b & 2 & 3 \end{vmatrix} = 0 \Rightarrow ab - 5b - 12a + 54 = 0$

If $D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 36 & a \\ 1 & b & 3 \end{vmatrix} = 0 \Rightarrow ab - 6a - 2b - 36 = 0$

If $D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & 36 \\ 1 & 2 & b \end{vmatrix} = 0 \Rightarrow b = 14$

For a = 8 & b= 14 $\Rightarrow$ D_1 & D_2 are also zero

For a = 8 & b = 14 $\Rightarrow$ D = D_1 = D_2 = D_3 = 0

$\Rightarrow$ infinitely many solutions.

7. The sum of all the real solutions of the equation $\log_{(x+3)}(6x^2 + 28x + 30) = 5 - 2\log_{(6x+10)}(x^2 + 6x + 9)$ is equal to :

(1) 2 (2) 1

(3) 0 (4) 4

Ans. **(3)**

Sol. $\log_{x+3}[(x + 3)(6x + 10)] = 5 - 2\log_{6x+10}(x + 3)^2$

$1 + \log_{x+3}(6x + 10) = 5 - 4\log_{6x+10}(x + 3)$

Let $\log_{(x+3)}(6x + 10) = A$

$\Rightarrow A + \frac{4}{A} = 4$ or $A = 2$

$\Rightarrow \log_{(x+3)}(6x + 10) = 2$

$\Rightarrow 6x + 10 = (x + 3)^2$

$\Rightarrow 6x + 10 = x^2 + 9 + 6x$

$\Rightarrow x^2 = 1, x = \pm 1$

So sum of roots = 0

8. If the points of intersection of the ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle of radius r and centre (a, b), then the value of $ab + 18r^2$ is

(1) 53 (2) 51

(3) 52 (4) 55

Ans. **(4)**

Sol. By family of curve equation of circle will be

$\Rightarrow S_1 + \lambda S_2 = 0$

$\Rightarrow (x^2 + 2y^2 - 6x - 12y + 23)$

$+ \lambda(4x^2 + 2y^2 - 20x - 12y + 35) = 0$

$\Rightarrow$ for circle coeff of x^2 = coeff. of y^2

$\Rightarrow \lambda = \frac{1}{2}$

So equation of circle is

$\Rightarrow x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0$

Centre $\left(\frac{8}{3}, 3\right)$: Radius $r = \sqrt{\frac{47}{18}} = r$

$\therefore ab + 18r^2 = 8 + 47 = 55$

9. Let PQ be a chord of the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$, perpendicular to the x-axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is :

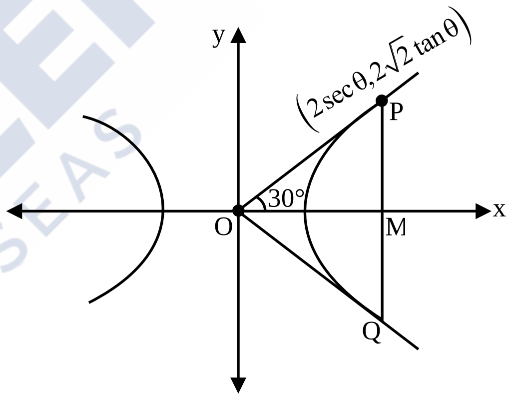
(1) $2\sqrt{3}$ (2) $\frac{8\sqrt{3}}{5}$

(3) $\frac{11}{5}$ (4) $\frac{9}{5}$

Ans. **(2)**

Sol. $e = \sqrt{1 + \frac{b}{4}} = \sqrt{3}$ $\Rightarrow b = 8$

$\therefore$ Hyperbola $\frac{x^2}{4} - \frac{y^2}{8} = 1$



$\frac{PM}{OM} = \tan 30°$

$\Rightarrow \frac{2\sqrt{2}\tan\theta}{2\sec\theta} = \frac{1}{\sqrt{3}} \Rightarrow \sin\theta = \frac{1}{\sqrt{6}}$

Area $= 2 \times \frac{1}{2} \times OM \times MP$

$= 2\sec\theta \times 2\sqrt{2}\tan\theta$

$= 4\sqrt{2}\frac{\sin\theta}{\cos^2\theta} = 4\sqrt{2} \times \frac{1}{\sqrt{6} \times \left(1 - \frac{1}{6}\right)} = \frac{8\sqrt{3}}{5}$

10. Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that $\vec{a}\times\vec{b}=2(\vec{a}\times\vec{c})$ .If $|\vec{a}|=1, |\vec{b}|=4$, $|\vec{c}|=2$, and the angle between $\vec{b}$ and $\vec{c}$ is 60°, then $|\vec{a}.\vec{c}|$ is :

(1) 2 (2) 4
(3) 0 (4) 1

Ans. **(4)**

Sol. $\vec{a}\times\vec{b}-2(\vec{a}\times\vec{c})=0$

$\vec{a}\times(\vec{b}-2\vec{c})=0 \Rightarrow \vec{b}-2\vec{c}=\lambda\vec{a}$ ...(1)

$|\lambda a|^2=|\vec{b}-2\vec{c}|^2 \Rightarrow \lambda^2|\vec{a}|^2=b^2+4c^2-4\vec{b}.\vec{c}$

$\lambda^2=16+16-4.4.2\cdot\frac{1}{2}$

$\lambda^2 = 16$

$\lambda = \pm 4$

$\because\ \vec{b}-2\vec{c}=\pm 4\vec{a}$

Dot with $\vec{c} \Rightarrow \vec{b}.\vec{c}-2|\vec{c}|^2=\pm 4(\vec{a}.\vec{c})$

$4.2.\frac{1}{2}-2.4=\pm 4(\vec{a}.\vec{c})$

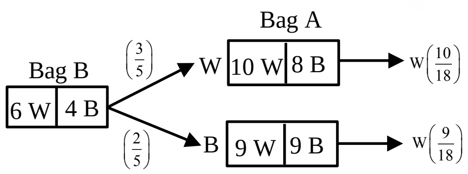
$|\vec{a}.\vec{c}|=1$

11. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A. Then a ball is randomly drawn from the bag A. If the probability, that the ball drawn is white, is $\frac{p}{q}$, gcd(p, q) = 1, then p + q is equal to

(1) 22 (2) 23
(3) 24 (4) 21

Ans. **(2)**

Sol.



$\therefore$ P(Drawn ball is white) $= \frac{3}{5}\times\frac{10}{18}+\frac{2}{5}\times\frac{9}{18}$

$= \frac{48}{90}=\frac{8}{15}=\frac{p}{q}$

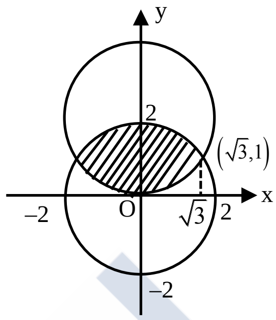
$\therefore\ p + q = 23$

12. The area of the region enclosed between the circles $x^2 + y^2 = 4$ and $x^2 + (y-2)^2 = 4$ is :

(1) $\frac{2}{3}(2\pi-3\sqrt{3})$ (2) $\frac{4}{3}(2\pi-3\sqrt{3})$

(3) $\frac{4}{3}(2\pi-\sqrt{3})$ (4) $\frac{2}{3}(4\pi-3\sqrt{3})$

Ans. **(4)**



Sol.

$A = 2\int_0^{\sqrt{3}}\left[\sqrt{4-x^2}-\left(2-\sqrt{4-x^2}\right)\right]dx$

$= 2\int_0^{\sqrt{3}}\left(2\sqrt{4-x^2}-2\right)dx$

$= 4\int_0^{\sqrt{3}}\left(\sqrt{4-x^2}-1\right)dx$

$= \left[4\left[\frac{1}{2}\left(x\sqrt{4-x^2}+4\sin^{-1}\frac{x}{2}\right)-x\right]\right]_0^{\sqrt{3}}$

$= 4\left[\frac{1}{2}\left(\sqrt{3}+4\times\frac{\pi}{3}\right)-\sqrt{3}\right]=4\left[\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right]$

$= \frac{8\pi}{3}-2\sqrt{3}$ (Sq. units)

13. The least value of $(\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2)$ is

(1) –1 (2) $4+\sqrt{10}$

(3) $4-\sqrt{10}$ (4) 1

Ans. **(3)**

Sol. $f(\theta) = \frac{1+\cos 2\theta}{2} - 3\sin 2\theta + 3\left(\frac{1-\cos 2\theta}{2}\right)+2$

$f(\theta) = 4 - 3\sin 2\theta - \cos 2\theta$

$f(\theta) \in [4-\sqrt{10}, 4+\sqrt{10}]$

14. Let $I(x) = \int \frac{3dx}{(4x+6)(\sqrt{4x^2+8x+3})}$ and $I(0) = \frac{\sqrt{3}}{4} + 20$. If $I\left(\frac{1}{2}\right) = \frac{a\sqrt{2}}{b} + c$, where a, b, c $\in$ N, gcd(a,b) = 1, then a + b + c is equal to :

(1) 29 (2) 28
(3) 31 (4) 30

Ans. **(3)**

Sol. Let $4x + 6 = \frac{1}{t} \Rightarrow x = \frac{\frac{1}{t} - 6}{4}$

$$4dx = -\frac{dt}{t^2} \qquad \left\{ x + 1 = \frac{\frac{1}{t} - 2}{4} \right.$$

$$\int \frac{3dx}{(4x+6)\sqrt{4(x+1)^2 - 1}}$$

$$\int \frac{3(-dt)}{4t^2 \times \frac{1}{t}\sqrt{4\left(\frac{\frac{1}{t}-2}{4}\right)^2 - 1}}$$

$$-\frac{3}{4}\int \frac{dt}{t\sqrt{\frac{(1-2t)^2}{4t^2} - 1}}$$

$$-\frac{3}{4}\int \frac{dt(2t)}{t\sqrt{1-4t}}$$

$$-\frac{3}{2}\int \frac{dt}{\sqrt{1-4t}} = -\frac{3}{2}\left(\frac{\sqrt{1-4t}}{\frac{1}{2} \times -4}\right) + c$$

$$= \frac{3}{4}\sqrt{1-4t} + c \qquad \because t = \frac{1}{4x+6}$$

$$= \frac{3}{4}\sqrt{1 - 4\left(\frac{1}{4x+6}\right)} + c$$

$$= \frac{3}{4}\sqrt{\frac{4x+6-4}{4x+6}} + c$$

$$I(x) = \frac{3}{4}\sqrt{\frac{4x+2}{4x+6}} + c$$

$$I(0) = \frac{3}{4}\sqrt{\frac{2}{6}} + c$$

$$I(0) = \frac{\sqrt{3}}{4} + c \Rightarrow c = 20$$

Hence $I(x) = \frac{3}{4}\sqrt{\frac{4x+2}{4x+6}} + 20$

$$I\left(\frac{1}{2}\right) = \frac{3}{4}\sqrt{\frac{4}{8}} + 20$$

$$= \frac{3}{4\sqrt{2}} + 20$$

$$= \frac{3\sqrt{2}}{8} + 20$$

a + b + c = 3 + 8 + 20 = 31

15. The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is

(1) 429 (2) 384
(3) 403 (4) 455

Ans. **(4)**

Sol. Let oranges are identical then

$x_1 + x_2 + x_3 + x_4 = 16$

and $x_1, x_2, x_3, x_4 \geq 1$

or $x'_1 + x'_2 + x'_3 + x'_4 = 12$

so total number of solutions are

$= {}^{12+3}C_3 = {}^{15}C_3 = 455$

16. Let A = {0, 1, 2,....9). Let R be a relation on A defined by (x, y) ∈ R if and only if |x–y| is a multiple of 3.

Given below are two statements:

Statement I: n(R) = 36

Statement II: R is an equivalence relation.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are correct

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are incorrect

Ans. **(2)**

Sol. Number of form 3K = 4

Number of form 3K + 1 = 3

Number of form 3K + 2 = 4

$4 \times 4 + 3 \times 3 + 3 \times 3 = 34$ relations

$\Rightarrow x\,R\,y \Rightarrow y\,R\,x$

$\Rightarrow (x - y) = 3\lambda, (y - z) = 3\mu$

$\Rightarrow (x - z) = 3(\lambda + \mu)$

R is reflexive, symmetric and transitive S_2 is ture

Ans. S_1 is false but S_2 is ture

17. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}, i = \sqrt{-1}$, then $(z^{201} - i)^8$ is equal to

(1) –1 (2) 0

(3) 1 (4) 256

Ans. **(4)**

Sol. $z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$$z^{201} = \cos\left(201\frac{\pi}{6}\right) + i\sin\left(201\frac{\pi}{6}\right) = -i$$

$(z^{201} - i)^8 = (-2i)^8 = 256$

18. If the mean and the variance of the data

Class	4–8	8–12	12–16	16–20
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda + \mu$ is

(1) 18 (2) 21

(3) 20 (4) 19

Ans. **(4)**

Sol. $$\mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{18 + 10\lambda + 56 + 126}{14 + \lambda}$$

$$= \frac{200 + 10\lambda}{\lambda + 14} = 10 + \left(\frac{60}{\lambda + 14}\right)$$

$\lambda + 14$ is multiply of 60 $\Rightarrow \lambda = 1$ or 6 or 16.

$$\sigma^2 = \frac{\sum x_i^2}{\lambda + 14} - (\mu)^2$$

$$\sigma^2 = \frac{6^2(3) + 10^2(\lambda) + 14^2(4) + (18)^2(7)}{\pi + 14} - \mu^2$$

for $\lambda = 6$, $\mu = 10 + 3 = 13$

$\lambda + \mu = 19$

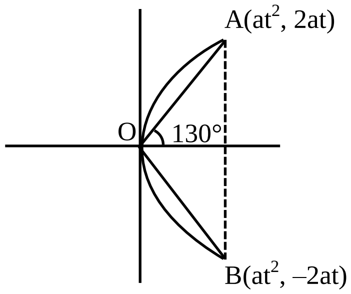
19. An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is

(1) $4(3 - \sqrt{3})$ (2) $2(8 - 3\sqrt{3})$

(3) $4(6 + \sqrt{3})$ (4) $2(3 + \sqrt{3})$

Ans. **(1)**

Sol.



$$M_{OA} = \frac{2t - 0}{t^2 - 0} = \frac{2}{f}$$

$$\frac{2}{t} = \tan 30°$$

$t = 2\sqrt{3}$

Req. Circle : $(x-12)^2 + y^2 = (4\sqrt{3})^2$

Least distance = |CP – R|

$= \left|2 - 4\sqrt{3} = 4\left(3 - \sqrt{3}\right)\right.$

20. Let $\sum_{k=1}^{n} a_k = \alpha n^2 + \beta n$. If $a_{10} = 59$ and $a_6 = 7a_1$ then $\alpha + \beta$ is equal to

(1) 12 (2) 3

(3) 5 (4) 7

Ans. **(3)**

Sol. $a_n = S_n - S_{n-1}$

$= (\alpha n^2 + \beta n) - (\alpha(n-1)^2 + \beta(n-1))$

(1) $a_{59} \Rightarrow 19\alpha + \beta = 59$

(2) $a_6 = 7a_1 \Rightarrow 11\alpha + \beta = 7(\alpha + \beta)$

$\Rightarrow 2\alpha = 3\beta$

$a = 3, \beta = 2$

$\boxed{\alpha + \beta = 5}$

SECTION-B

21. If the solution curve $y = f(x)$ of the differential equation $(x^2 - 4)\,y' - 2xy + 2x(4 - x^2)^2 = 0$ $x > 2$, passes through the point (3, 15), then the local maximum value of f is ………

Ans. (16)

Sol. $(x^2 - 4)y^1 - 2xy = -2x(x^2-4)^2$

$$d\left(\frac{y}{x^2-4}\right) = -2x$$

$y = (-x^2 + C)\,(x^2 - 4)$

for $x = 3$ $y = 15 \Rightarrow C = 12$

$y = (-x^2 + 12)\,(x^2 - 4)$

$y^1 = 0 \Rightarrow x = 2\sqrt{2}$

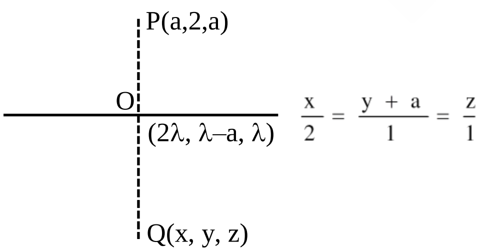
$$y_{\text{local maxi.}} = \left(\left(2\sqrt{2}\right)^2 - 4\right)\left(-\left(2\sqrt{2}\right)^2 + 12\right)$$

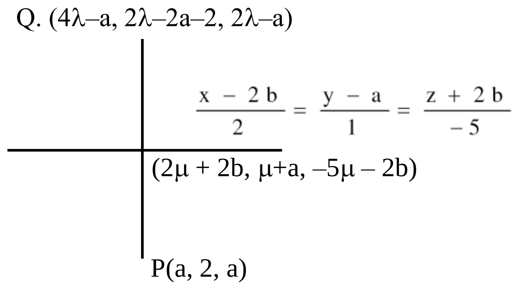
$= 16$

22. If the image of the point P(a, 2, a) in the line $\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$ is Q and the of image of Q in the line $\frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5}$ is P, then a + b is equal to………..

Ans. (3)

Sol.





$$\frac{2\lambda - a + a}{2} = -5\mu - 2b$$

and $\frac{a + 4\lambda - a}{2} = 2\mu + 2b$

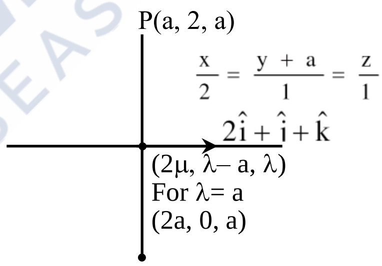
and $\frac{2\lambda - 2a - 2 + 2}{2} = \mu + a$

$\Rightarrow \lambda - \mu = b$

$\lambda - \mu = 2a$

$\lambda + 5\mu = -2b$

$\Rightarrow b = 2a$ and $\lambda = a$, $\mu = -a$



$$\left(a\hat{i} - 2\hat{j} + 0\hat{k}\right).\left(2\hat{i} + \hat{j} + \hat{k}\right) = 0$$

$2a - 2 = 0$

$a = 1$

$b = 2a = 2 \times 1 = 2$

$a + b = 1 + 2 = 3$

23. The number of elements in the set $S = \left\{x : x \in [0,100] \text{ and } \int_0^x t^2 \sin(x-t)dt = x^2\right\}$ is.....

Ans. (16)

Sol. $\int_0^x t^2 \sin(x-t)dt = x^2$

Use by parts

$-x^2\cos x + x^2 + 2x^2\cos x - 2x \sin x$

$-x^2\cos x + 2x \sin x + 2\cos x - 2 = x$

$\cos x = 1$

$x = 0, 2\pi, 4\pi, \ldots\ldots.,30\pi$

Total Elements = 16

24. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and B be a matrix such that $B(I - A) = I + A$. Then the sum of the diagonal elements of B^TB is equal to______.

Ans. **(3)**

Sol. $A^T = -A$

$B = (I + A)(I-A)^{-1}$

$B^T = ((I-A)^{-1})^T (I + A)^T$

$B^T = (I-A^T)^{-1} (I + A^T)$

$B^T = (I+A)^{-1} (I + A)$

$B^TB = (I+A)^{-1} (I - A)(I+A)(I-A)^{-1}$

$= (I+A)^{-1} (I + A)(I-A)(I-A)^{-1}$

$= I$

$tr(B^TB) = 3$

25. Let S denote the set of 4-digit numbers abcd such that $a > b > c > d$ and P denote the set of 5-digit numbers having product of its digits equal to 20. Then $n(S) + n(P)$ is equal to…………

Ans. **(260)**

Sol. For $n(s) = {}^{10}C_4 = 210$

$(5, 4, 1, 1, 1) \quad (5, 2, 2, 1, 1)$

For $n(p) = \frac{5!}{3!} + \frac{5!}{2!2!} = 50$

$n(s) + n(p) = 210 + 50 = 260$