JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON SATURDAY 24th JANUARY 2026) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f(x)=\int \frac{7x^{10}+9x^8}{(1+x^2+2x^9)^2}dx$, $x > 0$, $\lim\limits_{x\to 0} f(x)=0$ and $f(1)=\frac{1}{4}$. If $A=\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$ and $B = \text{adj}(\text{adj } A)$ be such that $|B| = 81$, then α^2 is equal to

(1) 2 (2) 3

(3) 1 (4) 4

Ans. **(4)**

Sol. $$f(x)=\frac{\int\left(\frac{7}{x^8}+\frac{9}{x^{10}}\right)}{\left(\frac{1}{x^9}+\frac{1}{x^7}+2\right)^2}dx$$

Put $t=\frac{1}{x^9}+\frac{1}{x^7}+2 \Rightarrow \frac{dt}{dx}=\frac{-9}{x^{10}}-\frac{7}{x^8}$

$$f(x)=\int\frac{-dt}{t^2}=\frac{1}{t}+C$$

$$f(x)=\frac{1}{\frac{1}{x^9}+\frac{1}{x^7}+2}+C$$

$$=\frac{x^9}{1+x^2+2x^9}+C$$

Given $f(1)=\frac{1}{4}=\frac{1}{4}+C \Rightarrow C=0$

$$f(x)=\frac{x^9}{1+x^2+2x^9}$$

$$f'(x)=\frac{(1+x^2+2x^9)-9x^8-x^9(2x+18x^8)}{(1+x^2+2x^9)^2}$$

$$f'(x)=\frac{36-20}{16}=1$$

$$A=\begin{pmatrix} 0 & 0 & 1 \\ 4 & 1 & 1 \\ \alpha^2 & \frac{1}{4} & 1 \end{pmatrix}$$

$|A| = |1 - \alpha^2| = 3$

$1 - \alpha^2 = 3, -3 \Rightarrow \alpha^2 = -2, 4$

Value of $\alpha^2 = 4$

$B = \text{adj}(\text{adj}A)$

$|B| = 81 = |A|^4 \Rightarrow |A| = 3$

2. Let the length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a > b)$, be 30. If its eccentricity is the maximum value of the function $f(t) = -\frac{3}{4} + 2t - t^2$, then $(a^2 + b^2)$ is equal to -

(1) 516 (2) 256

(3) 496 (4) 276

Ans. **(3)**

Sol. $f(t) = \frac{-3}{4}+2t-t^2$

$$f(t)\Big|_{\text{maximum}}=\frac{1}{4}=e \Rightarrow e^2=\frac{1}{16} \Rightarrow \frac{a^2-b^2}{a^2}=\frac{1}{16} \quad ..(1)$$

$$\because \frac{2b^2}{a}=30 \Rightarrow b^2=15a \ldots(2)$$

By (1) & (2)

$16(a^2 - 15a) = a^2 \Rightarrow 15a^2 - 16 \times 15a = 0$

$a = 16$

$b^2 = 240$

$a^2 + b^2 = 256 + 240$

$= 496$

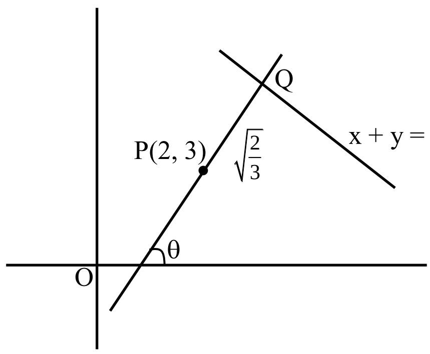
3. Let the angles made with the positive x-axis by two straight lines drawn from the point P(2, 3) and meeting the line x + y = 6 at a distance $\sqrt{\frac{2}{3}}$ from the point P be θ_1 and θ_2. Then the value of $(\theta_1 + \theta_2)$ is :

(1) $\frac{\pi}{12}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. (3)

Sol.



Let Q is $\left(\sqrt{\frac{2}{3}}\cos\theta+2, \sqrt{\frac{2}{3}}\sin\theta+3\right)$

so, $x+y=6$

$\sqrt{\frac{2}{3}}(\cos\theta + \sin\theta) + 5 = 6$

$\sin\theta + \cos\theta = \sqrt{\frac{3}{2}}$

$1 + \sin 2\theta = \frac{3}{2}$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{12}$ & $\frac{5\pi}{6}$

So $\theta_1 + \theta_2 = \frac{\pi}{2}$

4. Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}, \vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to

(1) $\frac{\sqrt{21}}{2}$ (2) 13

(3) $\frac{\sqrt{35}}{2}$ (4) 7

Ans. (Bonus)

NTA (3)

Sol. $\vec{v} = x\vec{a} + y\vec{b} = x(2\hat{i} - \hat{j} - \hat{k}) + y(\hat{i} + 3\hat{j} - \hat{k})$

$\vec{v} = (2x+y)\hat{i} + (3y-x)\hat{j} + (-x-y)\hat{k}$

$\left|\frac{\vec{v}.\vec{c}}{|\vec{c}|}\right| = \frac{1}{\sqrt{14}}$

$\vec{v}.\vec{c} = 2(2x+y) + 3y - x - 3x - 3y$

$= 2y$

$\left|\frac{2y}{\sqrt{14}}\right| = \frac{1}{\sqrt{14}} \Rightarrow |2y| = 1$

$|\vec{v}| = \sqrt{(2x+y)^2 + (3y-x)^2 + (x+y)^2}$

$= \sqrt{6x^2 + 11y^2 + 4xy - 6xy + 2xy}$

$= \sqrt{6x^2 + \frac{11}{4}} = \frac{\sqrt{24x^2+11}}{2}$

Now if we take $x^2 = 1$ then option $\frac{\sqrt{35}}{2}$ matches most probably NTA thought could been this.

5. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be an A.P. of four terms such that each term of the A. P. and its common difference l are integers. If $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 48$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4 + l^4 = 361$ then the largest term of the A.P. is equal to

(1) 27 (2) 24 (3) 21 (4) 23

Ans. (1)

Sol. a_1, a_2, a_3, a_4 as $a-3d, a-d, a+d, a+3d$

where $\boxed{d=\frac{\ell}{2}}$

$\because a_1+a_2+a_3+a_4=48 \Rightarrow 4a=48 \Rightarrow a=12$

& $a_1a_2a_3a_4+\ell^4=361 \Rightarrow (a^2-9d^2)(a^2-d^2)+16d^4$

$=361$

$\Rightarrow (144-9d^2)(144-d^2)+16d^4=361$

$\Rightarrow 25d^4-1440d^2+(144)^2=361$

$(5d^2-144)^2=19^2$

$\therefore 5d^2-144=19$ or -19

$d^2=\frac{163}{5}$ or $d^2=\frac{125}{5}=25$

$d=\sqrt{\frac{163}{5}}$ or $d=5$

$\therefore \ell=2\sqrt{\frac{163}{5}}$ or $\ell=10$

(rejected)

$\because$ common difference is an integer

$\therefore$ largest term $=12+15=27$

6. Let the image of parabola $x^2=4y$, in the line $x-y=1$ be $(y+\alpha)^2=b(x-c)$, $a, b, c \in \mathbb{N}$. Then $a+b+c$ is equal to

(1) 12 (2) 4

(3) 6 (4) 8

Ans. **(3)**

Sol. Parametric point P on $x^2=4y$ is $P(2t, t^2)$

$\therefore$ mirror image of P in $x-y=1$ is

$$Q \equiv \left(2t-\frac{2.1.(2t-t^2-1)}{2}, t^2+\frac{2.2(1).(2t-t^2-1)}{2}\right)$$

$Q \equiv (t^2+1, 2t-1) \equiv (h, k)$

$\therefore$ locus of Q is $x=\frac{(y+1)^2}{4}+1$ which is the required parabola.

$\therefore \boxed{(y+1)^2=4(x-1)}$

$\therefore a=1, b=4, c=1$

$\therefore \boxed{a+b+c=6}$

7. $$\left(\frac{1}{3}+\frac{4}{7}\right)+\left(\frac{1}{3^2}+\frac{1}{3}\times\frac{4}{7}+\frac{4^2}{7^2}\right)+\left(\frac{1}{3^3}+\frac{1}{3^2}\times\frac{4}{7}+\frac{1}{3}\times\frac{4^2}{7^2}+\frac{4^3}{7^3}\right)+ \ldots\ldots$$ upto infinite terms is equal to -

(1) $\frac{5}{2}$ (2) $\frac{7}{4}$ (3) $\frac{4}{3}$ (4) $\frac{6}{5}$

Ans. **(1)**

Sol. Let $a=\frac{4}{7}$, $b=\frac{1}{3}$

Multiply N^r and D^r by $(a-b)=\frac{4}{7}-\frac{1}{3}=\frac{5}{21}$

$$\frac{1}{a-b}[(a^2-b^2)+(a^3-b^3)+(a^4-b^4)+\ldots\infty]$$

$$\frac{1}{a-b}\left[\frac{a^2}{1-a}-\frac{b^2}{1-b}\right]=\frac{21}{5}\left[\frac{\frac{16}{49}}{1-\frac{4}{7}}-\frac{\frac{1}{9}}{1-\frac{1}{3}}\right]$$

$$=\frac{21}{5}\left[\frac{16}{21}-\frac{1}{6}\right]=\frac{21}{5}\left[\frac{96-21}{21.6}\right]$$

$$=\frac{75}{5.6}=\frac{15}{6}=\frac{5}{2}$$

8. Let $P=[p_{ij}]$ and $Q=[q_{ij}]$ be two square matrices of order 3 such that $q_{ij}=2^{(i+j-1)}p_{ij}$ and $\det(Q)=2^{10}$. Then the value of det(adj(adj P)) is :

(1) 32 (2) 16

(3) 81 (4) 124

Ans. **(2)**

Sol. $$\begin{vmatrix} 2p_{11} & 2^2p_{12} & 2^3p_{13} \\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23} \\ 2^3p_{31} & 2^4p_{32} & 2^5p_{33} \end{vmatrix}=2^{10}$$

$$2^2\cdot 2\cdot 2^3\begin{vmatrix} p_{11} & p_{12} & p_{13} \\ 2p_{21} & 2p_{22} & 2p_{23} \\ 2^2p_{31} & 2^2p_{32} & 2^2p_{33} \end{vmatrix}=2^{10}$$

$$2^9\begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}=2^{10} \Rightarrow |P|=2$$

$$|\operatorname{adj}(\operatorname{adj}(P))|=|P|^{(n-1)^2}=|P|^4=2^4=16$$

9. The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of the word "UDAYPUR" is :

(1) 1580 (2) 1578

(3) 1579 (4) 1581

Ans. (1)

Sol. ADIPRUU

$$A \rightarrow \frac{6!}{2!} = 360$$

$$D \rightarrow \frac{6!}{2!} = 360$$

$$P \rightarrow \frac{6!}{2!} = 360$$

$$R \rightarrow \frac{6!}{2!} = 360$$

UA $\rightarrow$ 5! = 120

UDAP $\rightarrow$ 3! = 6

UDAR $\rightarrow$ 3! = 6

UDAU $\rightarrow$ 3! = 6

UDAYPRU $\rightarrow$ 1

UDAYPUR $\rightarrow$ 1

Total = 1580

10. The largest value of n, for which 40^n divides 60!, is

(1) 13 (2) 11

(3) 12 (4) 14

Ans. (4)

Sol. $40^n = 2^{3n} \times 5^n$

$$E_2(60!) = \left[\frac{60}{2}\right] + \left[\frac{60}{2^2}\right] + \left[\frac{60}{2^3}\right] + \left[\frac{60}{2^4}\right] + \left[\frac{60}{2^5}\right]$$

= 30 + 15 + 7 + 3 + 1 = 56

$$E_5(60!) = \left[\frac{60}{5}\right] + \left[\frac{60}{5^2}\right]$$

= 12 + 2 = 14

$40^n = (2^3)^n \times 5^n = (2^3 \times 5)^n$

$60! = 2^{56} \times 5^{14} \ldots = 2^{14}.(2^3.5)^{14}$

$\therefore$ Maximum value of n is 14.

11. The sum of all values of α, for which the shortest distance between the lines

$\frac{x+1}{\alpha} = \frac{y-2}{-1} = \frac{z-4}{-\alpha}$ and $\frac{x}{\alpha} = \frac{y-1}{2} = \frac{z-1}{2\alpha}$ is $\sqrt{2}$, is

(1) 8 (2) –6

(3) 6 (4) –8

Ans. (2)

Sol. $$\sqrt{2} = \frac{\begin{vmatrix} -1 & 1 & 3 \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}\right|}$$

$$\sqrt{2} = \frac{-1(-2\alpha+2\alpha)-1(2\alpha^2+\alpha^2)+3(2\alpha+\alpha)}{\left|\hat{i}(-2\alpha+2\alpha)-\hat{j}(2\alpha^2+\alpha^2)+\hat{k}(2\alpha+\alpha)\right|}$$

$$\sqrt{2} = \frac{-3\alpha^2+9\alpha}{\sqrt{9\alpha^4+9\alpha^2}}$$

$$\sqrt{2} = \frac{-\alpha+3}{\sqrt{\alpha^2+1}}$$

$\Rightarrow 2\alpha^2 + 2 = \alpha^2 + 9 - 6\alpha$

$\alpha^2 + 6\alpha - 7 = 0$

$(\alpha + 7)(\alpha - 1) = 0$

$\alpha = -7, 1$

sum = –7 + 1 = – 6

option (2)

12. Let $f(\alpha)$ denote the area of the region in the first quadrant bounded by x = 0, x = 1, $y^2 = x$ and $y = |\alpha x - 5| - |1 - \alpha x| + \alpha x^2$. Then $(f(0) + f(1))$ is equal to

(1) 9 (2) 14

(3) 7 (4) 12

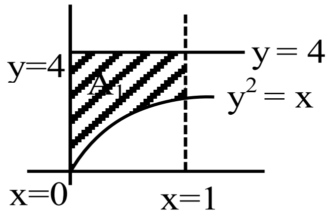
Ans. (3)

Sol. at $\alpha = 0 \Rightarrow f(0)$

$x = 0, x = 1, y^2 = x$

$y = |0\cdot x - 5| - |1 - 0\cdot x| + 0\cdot x^2$

y = 4



$$A_1 = \int_0^1 (4-\sqrt{x})dx$$

$$= 4x - \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$$

$$= 4 - \frac{2}{3}(1) = \frac{10}{3}$$

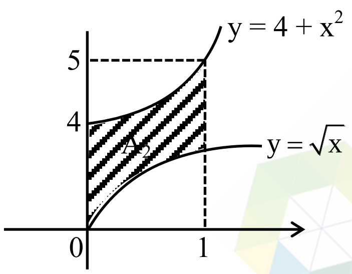
at $\alpha = 1 \Rightarrow f(1)$

$x = 0, x = 1, y^2 = x,$

$y = |x-5| - |1-x| + x^2$ in $x \in (0, 1)$

$y = 5 - x - (1-x) + x^2$

$y = 4 + x^2$



$$A_2 = \int_0^1 \left((4+x^2) - (\sqrt{x})\right)dx$$

$$= 4x + \frac{x^3}{3} - \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$$

$$= 4 + \frac{1}{3} - \frac{2}{3} = \frac{11}{3}$$

$$|f(0) + f(1)| = |A_1 + A_2| = \left|\frac{10}{3} + \frac{11}{3}\right| = \left|\frac{21}{3}\right| = 7$$

option (3)

13. If the domain of the function $f(x) = \sin^{-1}\left(\frac{1}{x^2-2x-2}\right)$, is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$, then $\alpha + \beta + \gamma + \delta$ is equal to

(1) 2 (2) 4

(3) 3 (4) 5

Ans. (2)

Sol. $-1 \le \frac{2}{x^2-2x-2} \le 1$

$$\frac{1+x^2-2x-2}{x^2-2x-2} \ge 0 \Rightarrow \frac{(x-1)^2-2}{(x-1)^2-3} \ge 0$$

$$\Rightarrow \frac{\left(x-1-\sqrt{2}\right)\left(x-1+\sqrt{2}\right)}{\left(x-1-\sqrt{3}\right)\left(x-1+\sqrt{3}\right)} \ge 0$$

$$x \in \left(-\infty, 1-\sqrt{3}\right) \cup \left[1-\sqrt{2}, 1+\sqrt{2}\right] \cup \left(1+\sqrt{3}, 0\right) \quad \ldots(1)$$

$$1 - \frac{1}{x^2-2x-2} \ge 0 \Rightarrow \frac{x^2-2x-3}{x^2-2x-2} \ge 0$$

$$\Rightarrow \frac{(x+1)(x-3)}{\left(x-1+\sqrt{3}\right)\left(x-1-\sqrt{3}\right)} \ge 0$$

$$x \in (-\infty, -1] \cup \left(1-\sqrt{3}, \sqrt{3}+1\right) \cup [3, \infty) \quad \ldots(2)$$

$(1) \cap (2)$

$$\Rightarrow x \in (-\infty, -1] \cup \left[1-\sqrt{2}, 1+\sqrt{2}\right] \cup [3, \infty)$$

$$\therefore \alpha + \beta + \gamma + \delta = 4$$

14. Let $X = \{x \in \mathbb{N} : 1 \le x \le 19\}$ and for some $a, b \in \mathbb{R}$, $Y = \{ax + b : x \in X\}$. If the mean and variance of the elements of Y are 30 and 750, respectively, then the sum of all possible values of b is

(1) 20 (2) 80

(3) 100 (4) 60

Ans. (4)

Sol. $\Sigma y_i = a\Sigma x_i + \Sigma b$

$= a \times (1 + 2 + \ldots\ldots + 19) + 19b$

$$\frac{\Sigma y_i}{19} = \frac{a \times 19 \times 20}{2 \times 19} + b$$

$30 = 10a + b \quad \ldots\ldots(1)$

Variance of $X = \frac{\Sigma x_i^2}{19} - \left(\frac{\Sigma x_i}{19}\right)^2$

$= \frac{19\times 20\times 39}{19\times 6} - (10)^2 = 30$

Variance of Y = a^2 (variance of X)

$750 = a^2 \times 30$

$a^2 = 25 \Rightarrow a = \pm 5$

if $a = +5 \Rightarrow b = 30 - 50 = -20$ ...from (i)

if $a = -5 \Rightarrow b = 30 + 50 = 80$ ...from (i)

sum of values of b = $80 - 20 = 60$

options (4)

15. Consider the following three statements for the function $f : (0, \infty) \to \mathbb{R}$ defined by

$f(x) = |\log_e x| - |x - 1|$:

(I) f is differentiable at all $x > 0$.

(II) f is increasing in (0, 1).

(III) f is decreasing in $(1, \infty)$.

Then.

(1) All (I), (II) and (III) are TRUE.

(2) Only (I) is TRUE.

(3) Only (II) and (III) are TRUE.

(4) Only (I) and (III) are TRUE.

Ans. **(4)**

Sol. $f(x) = |\ell nx| - |x - 1|$

$$= \begin{cases} \ell nx - (x-1) & x \geq 1 \\ -\ell nx + (x-1) & 0 < x < 1 \end{cases}$$

$$= \begin{cases} \ell nx - x + 1 & x \geq 1 \\ -\ell nx + x - 1 & 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{x} - 1 & x \geq 1 \\ -\frac{1}{x} + 1 & 0 < x < 1 \end{cases}$$

$f'(1^+) = f'(1^-) = 0 \Rightarrow f(x)$ is differentiable $\forall\ x > 0$

$f'(x) < 0 \quad \forall x > 1$

$f'(x) < 0 \quad \forall 0 < x < 1$

$\Rightarrow f(x)$ is decreasing $\forall\ x \in (0, \infty)$

Option (4)

16. Let y = y (x) be a differentiable function in the interval $(0, \infty)$ such that $y(1) = 2$.

and $\lim_{t \to x}\left(\frac{t^2 y(x) - x^2 y(t)}{x - t}\right) = 3$ for each $x > 0$.

Then 2y(2) is equal to

(1) 18

(2) 23

(3) 27

(4) 12

Ans. **(2)**

Sol. $\lim_{t\to x}\frac{2t\,f(x) - x^2 f'(t)}{-1} = 3$

$x^2 f'(x) - 2x\,f(x) = 3$

$\frac{dy}{dx} - \frac{2y}{x} = \frac{3}{x^2}$

I.F. $= e^{-\int \frac{2}{x}dx} = e^{-2\log_e x} = 1/x^2$

$y \cdot \frac{1}{x^2} = \int \frac{3}{x^4}dx$

$\frac{y}{x^2} = -\frac{1}{x^3} + c \Rightarrow y = cx^2 - \frac{1}{x} = f(x)$

$f(1) = 2 = c - 1 \Rightarrow c = 3$

$f(x) = 3x^2 - \frac{1}{x}$

$f(2) = 12 - \frac{1}{2} \Rightarrow 2f(2) = 23$

17. Let f be a function such that $3f(x) + 2f\left(\frac{m}{19x}\right) = 5x$, $x \neq 0$, where $m = \sum_{i=1}^{9} (i)^2$. Then $f(5) - f(2)$ is equal to

(1) –9 (2) 36

(3) 18 (4) 9

Ans. **(3)**

Sol. $m = \frac{9\times10\times19}{6} = 15\times19$

$3f(x) + 2f\left(\frac{15}{x}\right) = 5x$

Replace x by $\frac{15}{x}$

$3f\left(\frac{15}{x}\right) + 2f(x) = \frac{75}{x}$

$9f(x) - 4f(x) = 15x - \frac{150}{x}$

$5f(x) = 15x - \frac{150}{x}$

$f(x) = 3x - \frac{30}{x}$

$f(5) = 15 - \frac{30}{5} = 9$

$f(2) = 6 - 15 = -9$

$f(5) - f(2) = 18$

18. The smallest positive integral value of a, for which all the roots of $x^4 - ax^2 + 9 = 0$ are real and distinct, is equal to

(1) 9 (2) 3

(3) 4 (4) 7

Ans. (4)

Sol. $x^4 - ax^2 + 9 = 0$ …(1)

let $x^2 = t$

$t^2 - at + 9 =$ …(2)

for roots of equation (1) to be real & distinct roots of equation (2) must be positive & distinct.

(i) $D > 0 \Rightarrow a^2 - 36 > 0 \Rightarrow a \in (-\infty, -6) \cup (6, \infty)$

(ii) $\frac{-b}{2a} > 0 \Rightarrow \frac{a}{2} > 0 \Rightarrow a > 0$

(iii) $f(0) > 0 \Rightarrow 9 > 0 \Rightarrow a \in R$

By (i) $\cap$ (ii) $\cap$ (iii)

$\therefore a \in (6, \infty)$

$\therefore$ least integral value of a is 7

19. Let $\vec{a} = 2\hat{i} - 5\hat{j} + 5k$ and $\vec{b} = \hat{i} - \hat{j} + 3k$. If $\vec{c}$ is a vector such that $2\left(\vec{a}\times\vec{c}\right) + 3\left(\vec{b}\times\vec{c}\right) = \vec{0}$ and $\left(\vec{a}-\vec{b}\right).\vec{c} = -97$, then $\left|\vec{c}\times k\right|^2$ is equal to

(1) 193 (2) 233

(3) 218 (4) 205

Ans. (3)

Sol. $2(\vec{a}\times\vec{c}) + 3(\vec{b}\times\vec{c}) = 0$

$\Rightarrow (2\vec{a}+3\vec{d})\times\vec{c} = 0 \Rightarrow \vec{c} = \lambda(2\vec{a}+3\vec{d})$

$\Rightarrow \vec{c} = \lambda\,(7i - 13j + 19k)$

Now $(\vec{a}-\vec{b})\cdot\vec{c} - = \lambda\,(7+52+38) + 97\lambda = -97$

$\Rightarrow \lambda = -1$

Now $\vec{c} = -7i + 13j - 19k$

$\Rightarrow \vec{c}\times\vec{k} - = 7j + 13i \Rightarrow |\vec{c}\times\hat{k}|^2 = 7^2 + 13^2 = 218$

20. Let [t] denote the greatest integer less than or equal to t. If the function

$$f(x) = \begin{cases} b^2 \sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\cos x + \sin x)\cos x\right]\right) & ,x<0 \\ \dfrac{\sin x - \frac{1}{2}\sin 2x}{x^3} & ,x>0 \\ a & ,x=0 \end{cases}$$

is continuous at x = 0, then $a^2 + b^2$ is equal to

(1) $\frac{5}{8}$ (2) $\frac{9}{16}$

(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. (3)

Sol. $f(0) = a$

$$\text{RHL} = \lim_{x\to0^+}\frac{\sin x(1-\cos x)}{x^3} = \frac{1}{2}$$

$$\text{LHL} = \lim_{x\to0^-}\left(b^2\sin\frac{\pi}{2}\left[\frac{\pi}{2}(\sin x + \cos x)\cos x\right]\right) = b^2$$

$\therefore\ a = \frac{1}{2}$ & $b^2 = \frac{1}{2}$

so $(a^2 + b^2) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

SECTION-B

21. If f(x) satisfies the relation $f(x) = e^x + \int_0^1 \left(y + xe^x\right) f(y)\,dy$, then e + f(0) is equal to ________.

Ans. **(2)**

Sol. $f(x) = e^x + \int_0^1 yf(y)dy + xe^x \int_0^1 f(y)dy$

$f(x) = e^x + A + Bxe^x$

$A = \int_0^1 yf(y)\,dy = \int_0^1 y(A + e^y + By\,e^y)\,dy$

$A = \frac{A}{2} + (0 - (-1)) + B(e - 1)$

$\frac{A}{2} + B(1 - e) = 1$

$B = \int_0^1 f(y)\,dy$

$B = \int_0^1 (e^y + A + By\,e^y)dy$

$B = (e - 1) + A + B(0 - (-1))$

$B = e - 1 + A + B \Rightarrow A = 1 - e$

$f(x) = e^x + A + Bxe^x$

$f(0) = 1 + A = 1 - e + 1 = 2 - e$

$e + f(0) = 2$

22. Let (h, k) lie on the circle C : $x^2 + y^2 = 4$ and the point (2h + 1, 3k + 2) lie on an ellipse with eccentricity e. Then the value of $\frac{5}{e^2}$ is equal to ____________.

Ans. **(9)**

Sol. Let $P \equiv (2\cos\theta, 2\sin\theta)$

$\therefore$ coordinates of $Q = (4\cos\theta + 1, 6\sin\theta + 3)$

$\therefore$ locus of Q is $\left(\frac{x-1}{4}\right)^2 + \left(\frac{y-3}{6}\right)^2 = 1$

$\therefore\ e^2 = 1 - \frac{16}{36} = \frac{5}{9}$

$\therefore\ \boxed{\frac{5}{e^2} = 9}$

23. Let z = (1 + i) (1 + 2i) (1 + 3i).... (1 + ni), where $i = \sqrt{-1}$. If $|z|^2 = 44200$, then n is equal to –

Ans. **(5)**

Sol. $|z|^2 = 2^3.5^2.13.17$

$\prod_{r=1}^{n}(1 + r^2) = 2^3.5^2.13.17 = (2).(5).(2.5).(17).(2.13) = 2.5.10.17.26$

so n = 5

24. Let S be a set of 5 elements and P(S) denote the power set of S. Let E be an event of choosing an ordered pair (A, B) from the set P(S) × P(S) such that $A \cap B = \varnothing$. If the probability of the event E is $\frac{3^p}{2^q}$, where p, q $\in \mathbb{N}$, then p + q is equal to

Ans. **(15)**

Sol. S = {a, b, c, d, e}

P(S) contains 32 elements

both set A and set B are subsets of P(S)

Every element has 4 choices

A	B
✓	✓
✓	x
x	✓
x	x

Favourable cases = 3^5

Total cases = 4^5

$P = \frac{3^5}{4^5} = \frac{3^5}{2^{10}}$

m = 5, n = 10

m + n = 15

25. The number of elements in the set $\left\{x \in [0, 180°] : \tan(x + 100°) = \tan(x + 50°)\tan x \tan(x - 50°)\right\}$ is______.

Ans. **(4)**

Sol. $\frac{\tan(x + 100°)}{\tan x} = \tan(x + 50°)\tan(x - 50°)$

$\frac{\sin(x + 100°)\cos x}{\cos(x + 100°)\sin x} = \frac{\sin(x + 50°)\sin(x - 50°)}{\cos(x + 50°)\cos(x - 50°)}$

Apply C & D

$\frac{\sin(2x + 100°)}{\sin 100°} = \frac{\cos 100°}{-\cos 2x}$

$2\sin(2x + 100°)\cos 2x + \sin 200° = 0$

$\sin(4x + 100°) + \sin 100° + \sin 200° = 0$

$\sin(4x + 100°) = -2\sin 150° \cos 50°$

$\sin(4x + 100°) = -\cos 50° = \sin(-40°)$

$\therefore\ 4x + 100° = n\pi + (-1)^n \cdot (-40°)$

$x = \frac{n\pi + (-1)^{n+1}(40°) - 100°}{4}$

$\therefore$ x = 30°, 55°, 120°, 145° in (0, π)

$\therefore$ no. of solutions = 4