JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON SATURDAY 24th JANUARY 2026) **TIME : 9:00 AM TO 12:00 Noon**

MATHEMATICS | TEST PAPER WITH SOLUTIONS

SECTION-A

1. If the function f(x)

$$= \frac{e^x\left(e^{\tan x - x} - 1\right) + \log_e(\sec x + \tan x) - x}{\tan x - x} \text{ is}$$

Continuous at x = 0, then the value of f(0) is equal to

(1) 2 (2) $\frac{2}{3}$

(3) $\frac{1}{2}$ (4) $\frac{3}{2}$

Ans. (4)

Sol. $f(0) = \lim\limits_{x\to 0} \frac{e^{\tan x} - e^x + \ell n\left(\sec x + \tan x\right) - x}{\tan x - x}$

Applying L'hospital rule

$\Rightarrow f(0) = \lim\limits_{x\to 0} \frac{e^{\tan x}.\sec^2 x - e^x + \sec x - 1}{\sec^2 x - 1}$

$\Rightarrow f(0) = \lim\limits_{x\to 0} \frac{e^{\tan x}(\sec^2 x - 1) + (e^{\tan x} - e^x) + \sec x - 1}{\tan^2 x}$

$\Rightarrow f(0) = \lim\limits_{x\to 0} \left(e^{\tan x} + \frac{e^x\left(e^{\tan x - x} - 1\right)}{\tan^2 x} + \frac{1}{\sec x + 1}\right)$

$\Rightarrow f(0) = 1 + 0 + \frac{1}{2} = \frac{3}{2}$

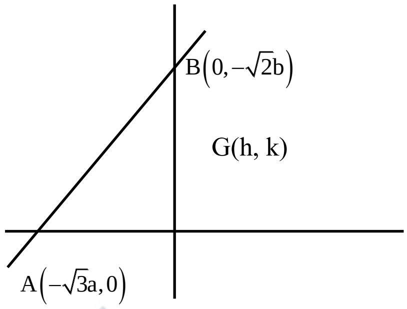
2. Let a circle of radius 4 pass through the origin O, the points A $\left(-\sqrt{3}a, 0\right)$ and B$\left(0, -\sqrt{2}\,b\right)$, where a and b are real parameters and ab ≠ 0. Then the locus of the centroid of ΔOAB is a circle of radius

(1) $\frac{5}{3}$ (2) $\frac{7}{3}$

(3) $\frac{8}{3}$ (4) $\frac{11}{3}$

Ans. (3)

Sol.



AB = 8

$3a^2 + 2b^2 = 64$

Centroid G(h, k)

$h = -\frac{\sqrt{3}a}{3}$, $k = -\frac{\sqrt{2}b}{3}$

$a = -\sqrt{3}h$, $b = \frac{-3}{\sqrt{2}}k$

$9h^2 + 9k^2 = 64$

$x^2 + y^2 = \frac{64}{9}$

$r = \frac{8}{3}$

3. Let the lines L_1: $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$, $\lambda \in \mathbb{R}$ and L_2 : $\vec{r} = \left(4\hat{i} + \hat{j}\right) + \mu\left(5\hat{i} + 2\hat{j} + \hat{k}\right)$, $\mu \in \mathbb{R}$, intersect at the point R. Let P and Q be the points lying on lines L_1 and L_2, respectively, such that $\left|\overrightarrow{PR}\right| = \sqrt{29}$ and $\left|\overrightarrow{PQ}\right| = \sqrt{\frac{47}{3}}$. If the point P lies in the first octant, then $27(QR)^2$ is equal to

(1) 340 (2) 360

(3) 320 (4) 348

Ans. **(2)**

Sol. For POI

$2\lambda + 1 = 5\mu + 4$; $3\lambda + 2 = 2\mu + 1$; $4\lambda + 3 = \mu$

$\Rightarrow \lambda = \mu = -1$

$R(-1, -1, -1)$ $\quad P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

$PR^2 = 29 \Rightarrow (2\lambda + 2)^2 + (3\lambda + 3)^2 + (4\lambda + 4)^2 = 29$

$\Rightarrow \lambda = 0$ or $\lambda = -2$ (Reject)

$\Rightarrow P(1, 2, 3)$

$Q(5\mu + 4, 2\mu + 1, \mu)$

$|PQ| = \sqrt{\frac{47}{3}} \Rightarrow PQ^2 = \frac{47}{3}$

$\Rightarrow (5\mu + 3)^2 + (2\mu - 1)^2 + (\mu - 3)^2 = \frac{47}{3}$

$\Rightarrow \mu = -\frac{1}{3}$

$Q = \left(\frac{7}{3}, \frac{1}{3}, -\frac{1}{3}\right)$

$(QR)^2 = \left(\frac{7}{3}+1\right)^2 + \left(\frac{1}{3}+1\right)^2 + \left(-\frac{1}{3}+1\right)^2$

$= \frac{100+16+4}{9} = \frac{120}{9}$

$\Rightarrow 27 \times (QR)^2 = 27 \times \frac{120}{9} = 360$

4. Let 729, 81, 9, 1, …. be a sequence and P_n denote the product of the first n terms of this sequence.

If $2\sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = \frac{3^\alpha - 1}{3^\beta}$ and gcd $(\alpha, \beta) = 1$, then $\alpha + \beta$ is equal to

(1) 73 $\qquad$ (2) 74

(3) 75 $\qquad$ (4) 76

Ans. **(1)**

Sol. $P_n = 729.81.9$ …….(n terms)

$= 3^6.3^4.3^2$ …….. 3^{-2n+8}

$P_n = 3^{6+4+2+\ldots\ldots+(-2n+8)} = 3^{n(7-n)}$

$P_n^{1/n} = 3^{7-n}$

$\Rightarrow \sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = 3^6 + 3^5 + \ldots.. + (40\,\text{terms})$

$= 3^6\left[\frac{1-\left(\frac{1}{3}\right)^{40}}{1-\frac{1}{3}}\right]$

$= \frac{3^6\left[3^{40}-1\right]\times 3^1}{3^{40}\times 2}$

$\sum(P_n)^{\frac{1}{n}} = \frac{(3^{40}-1)}{2\times 3^{33}}$, $\qquad \alpha = 40$

$\beta = 33$

$\alpha + \beta = 73$

5. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a}\times\vec{b}$. Let $\vec{d}$ be a vector such that $|\vec{d} - \vec{a}| = \sqrt{11}, |\vec{c}\times\vec{d}| = 3$ and the angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$. Then $\vec{a}\cdot\vec{d}$ is equal to

(1) 11

(2) 3

(3) 0

(4) 1

Ans. **(3)**

Sol. $\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$\vec{c} = 2\hat{i} + 2\hat{k} + \hat{k}$, $|c| = 3$

$|\vec{c}\times\vec{d}| = 3$

$|\vec{c}||\vec{d}|\sin\frac{\pi}{4} = 3 \Rightarrow |\vec{d}| = \sqrt{2}$

$|\vec{d} - \vec{a}| = \sqrt{11}$

$\Rightarrow |\vec{a}|^2 + |\vec{d}|^2 - 2\vec{a}.\vec{d} = 11$

$9 + 2 - 2\vec{a}.\vec{d} = 11$

$\vec{a}.\vec{d} = 0$

6. If the domain of the function

$$f(x) = \log_{(10x^2-17x+7)}(18x^2 - 11x + 1)$$

is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then $90(a + b + c + d + e)$ equals:

(1) 170

(2) 177

(3) 307

(4) 316

Ans. **(4)**

Sol. $18x^2 - 11x + 1 > 0$

$(2x - 1)(9x - 1) > 0$

$x < \frac{1}{9}$ or $\frac{1}{2} < x$

Also $10x^2 - 17x + 7 > 0$

$(x - 1)(10x - 7) > 0$

$x < \frac{7}{10}$ or $1 < x$

& $10x^2 - 17x + 7 \neq 1$

$$x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \frac{7}{10}\right) \cup (1, \infty) - \left\{\frac{6}{5}\right\}$$

$$90(a+b+c+d+e) = 90\left(\frac{1}{9} + \frac{1}{2} + \frac{7}{10} + 1 + \frac{6}{5}\right)$$

$= 10 + 45 + 63 + 90 + 108 = 316$

7. Let each of the two ellipses $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, $(A < B)$ have eccentricity $\frac{4}{5}$. Let the lengths of the latus recta of E_1 and E_2 be ℓ_1 and ℓ_2, respectively, such that $2\ell_1^2 = 9\ell_2$. If the distance between the foci of E_1 is 8, then the distance between the foci of E_2 is

(1) $\frac{96}{5}$ (2) $\frac{32}{5}$

(3) $\frac{16}{5}$ (4) $\frac{8}{5}$

Ans. **(2)**

Sol. $2ae = 8 \Rightarrow a = 5$

$b^2 = a^2 \ (1 - e^2)$

$b^2 = a^2 \times \frac{9}{25} \quad b^2 = 9$

$$E_1 : \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\ell_1 : \frac{2b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$$

$$A^2 = B^2\left(1 - e^2\right) \Rightarrow A^2 = \frac{9}{25}B^2 \Rightarrow A = \frac{3}{5}B$$

$$2\ell^2 = 9\ell_2 \Rightarrow 2\left(\frac{18}{5}\right)^2 = 9\ell_2 \Rightarrow \ell_2 = \frac{4 \times 18}{25}$$

$$\frac{2A^2}{B} = \frac{72}{25} \Rightarrow A^2 \frac{36}{25}B$$

$$\frac{9}{25}B^2 = \frac{36B}{25} \Rightarrow B = 4,$$

Distance between focii $2Be = 2 \times \frac{4}{5} \times 4 = \frac{32}{5}$

8. The value of $\dfrac{\sqrt{3}\operatorname{cosec}20° - \sec 20°}{\cos 20° \cos 40° \cos 60° \cos 80°}$ is equal to

(1) 32 (2) 16

(3) 64 (4) 12

Ans. **(3)**

Sol.
$$E = \frac{\frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}}{\frac{1}{2} \cdot \frac{1}{4} \cdot \cos 60°}$$

$$= \frac{\left(\sqrt{3}\cos 20° - \sin 20°\right)}{\cos 20° . \sin 20°} 16$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right) 32 \times 2}{2\cos 20° . \sin 20°}$$

$$= \frac{\sin 40°}{\sin 40°} \times 64 = 64$$

9. Let $S=\left\{z\in\mathbb{C}:\left|\frac{z-6i}{z-2i}\right|=1 \text{ and } \left|\frac{z-8+2i}{z+2i}\right|=\frac{3}{5}\right\}$.

Then $\sum_{z\in S}|z|^2$ is equal to

(1) 398 (2) 413

(3) 423 (4) 385

Ans. (4)

Sol. Solving $\left|\frac{z-6i}{z-2i}\right|=1 \Rightarrow y=4$ ..... (1)

(where $z = x + iy$)

Now solving $\left|\frac{z-8+2i}{z+2i}\right|=\frac{3}{5}$

$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0$ .... (2)

Solving (1) & (2) $\Rightarrow z = 17 + 4i$ & $8 + 4i$

$\Rightarrow \Sigma |z|^2 = (17)^2 + (4)^2 + (8)^2 + (4)^2 = 385$

10. If $\cot x = \frac{5}{12}$ for some $x\in\left(\pi,\frac{3\pi}{2}\right)$, then

$\sin 7x\left(\cos\frac{13x}{2}+\sin\frac{13x}{2}\right)+$

$\cos 7x\left(\cos\frac{13x}{2}-\sin\frac{13x}{2}\right)$ is equal to

(1) $\frac{4}{\sqrt{26}}$ (2) $\frac{6}{\sqrt{26}}$

(3) $\frac{1}{\sqrt{13}}$ (4) $\frac{5}{\sqrt{13}}$

Ans. (3)

Sol. $\cot x=\frac{5}{12}\Rightarrow \cos x=\frac{-5}{13}=2\cos^2\frac{x}{2}-1$

$\cos\left(\frac{x}{2}\right)=-\frac{2}{\sqrt{13}}$ or $\frac{2}{\sqrt{13}}$ (rejected)

$\left\{\because \frac{x}{2}\in\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\right\}$

$\left(\sin 7x\frac{\sin 13x}{2}+\cos 7x\frac{\cos 13x}{2}\right)+\left(\sin 7x\frac{\cos 13x}{2}-\cos 7x\frac{\sin 13x}{2}\right)$

$\cos\left(7x-\frac{13x}{2}\right)+\sin\left(7x-\frac{13x}{2}\right)$

$\cos\frac{x}{2}+\sin\left(\frac{x}{2}\right)$

$\frac{3}{\sqrt{13}}-\frac{2}{\sqrt{13}}=\frac{1}{\sqrt{13}}$

11. Let $f(t)=\int\left(\frac{1-\sin(\log_e t)}{1-\cos(\log_e t)}\right)dt, t>1$.

If $f(e^{\pi/2}) = -e^{\pi/2}$ and $f(e^{\pi/4}) = \alpha\, e^{\pi/4}$, then α equals

(1) $-1-\sqrt{2}$

(2) $-1-2\sqrt{2}$

(3) $1+\sqrt{2}$

(4) $-1+\sqrt{2}$

Ans. (1)

Sol. $f(t)=\int\frac{1-\sin(\ln t)}{1-\cos(\ln t)}dt$

Let $\ln t = x \Rightarrow t = e^x \Rightarrow dt = e^x\, dx$

$=\frac{1}{2}\int\left(\operatorname{cosec}^2\frac{x}{2}-2\cot\frac{x}{2}\right)e^x dx - t\cot\left(\frac{\ln t}{2}\right)+C$

$\left(\therefore \int(f(x)+f'(x))e^x dx = f(x).e^x + c\right)$

Now $f\left(e^{\pi/2}\right)=-e^{\pi/2}\cot\left(\frac{\pi}{4}\right)+C=-e^{\frac{\pi}{2}}$ (given)

$\Rightarrow C = 0$

Now $f\left(e^{\pi/4}\right)=-e^{\pi/4}\cot\left(\frac{\pi}{8}\right)+C=-e^{\frac{\pi}{4}}(\sqrt{2}+1)$

12. Let R be a relation defined on the set $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ by

$R = \{((a, b), (c, d)) : 2a + 3b = 3c + 4d\}$.

Then the number of elements in R is

(1) 6 (2) 18

(3) 12 (4) 15

Ans. (3)

Sol.

(a, b)	(c, d)
(1, 1)	x
(1, 2)	x
(1, 3)	(1, 2)
(1, 4)	(2, 2)
(2, 1)	(1, 1)
(2, 2)	(2, 1)
(2, 3)	(3, 1)
(2, 4)	(4, 1)
(3, 1)	x
(3, 2)	x
(3, 3)	(1, 3)
(3, 4)	(2, 3)
(4, 1)	(1, 2)
(4, 2)	(2, 2)
(4, 3)	(3, 2)
(4, 4)	(4, 2)

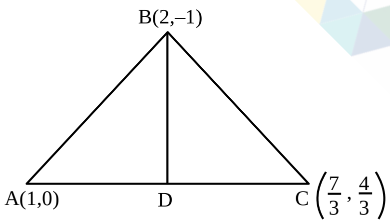
13. Let A(1, 0), B(2, –1) and $C\left(\frac{7}{3},\frac{4}{3}\right)$ be three points. If the equation of the bisector of the angle ABC is $\alpha x+\beta y=5$, then the value of $\alpha^2+\beta^2$ is

(1) 8 (2) 5

(3) 13 (4) 10

Ans. **(4)**

Sol.



$$\frac{BD}{DC}=\frac{AB}{AC}=\frac{\sqrt{2}\times 3}{5\sqrt{2}}=\frac{3}{5}$$

$$D=\left(\frac{12}{8},\frac{4}{8}\right)=\left(\frac{3}{2},\frac{1}{2}\right)$$

$$\text{Slope of AD}=\frac{-3/2}{\frac{1}{2}}=-3$$

$3x+y=5$

$\alpha=3,\beta=1;\ \alpha^2+\beta^2=10$

14. Let $S=\frac{1}{25!}+\frac{1}{3!23!}+\frac{1}{5!21!}+\ldots$ up to 13 terms.

If $13S=\frac{2^k}{n!}$, $k\in N$, then

$n+k$ is equal to

(1) 51

(2) 52

(3) 49

(4) 50

Ans. **(3)**

Sol. $\frac{1}{26!}\left(\frac{26!}{25!1!}+\frac{26!}{3!23!}+\frac{26!}{5!21!}+\ldots..+13\text{ terms}\right)$

$\frac{1}{26!}\left({}^{26}C_1+{}^{26}C_3+{}^{26}C_5+\ldots.+13\,\text{terms}\right)$

$\frac{1}{26!}\left({}^{26}C_1+{}^{26}C_5+\ldots.+{}^{26}C_{25}\right)$

$\Rightarrow S=\frac{1}{26!}\times 2^{25}$

$\Rightarrow 13S=\frac{2^{24}}{25!}$

so $n+k=25+24=49$

15. Consider an A.P.: $a_1, a_2,\ldots,a_n$; $a_1>0$. If $a_2-a_1=\frac{-3}{4}$, $a_n=\frac{1}{4}a_1$, and

$\sum_{i=1}^{n}a_i=\frac{525}{2}$, then $\sum_{i=1}^{17}a_i$ is equal to

(1) 476 (2) 952

(3) 238 (4) 136

Ans. **(3)**

ALLEN OVERSEAS

Sol. $S_n = \frac{n}{2}[a_1 + a_n] = \frac{525}{2}$, $d = \frac{-3}{4}$

$$\frac{n}{2}\left[a_1 + \frac{a_1}{4}\right] = \frac{525}{2}$$

$$\frac{5a_1 n}{4} = 525$$

$$a_1 n = 420$$

$$a_n = a_1 + (n-1)\left(\frac{-3}{4}\right)$$

$$\Rightarrow \frac{-3}{4}a_1 = \left(\frac{-3}{4}\right)(n-1) \Rightarrow a_1 = n-1$$

$n(n-1) = 420$

$n^2 - n - 420 = 0$

$(n-21)(n+20) = 0$

$n = 21,\ a_1 = 20$

$$\sum_{i=1}^{17} a_i = \frac{17}{2}[2a_1 + 16d]$$

$$= \frac{17}{2}\left[40 + 16\left(\frac{-3}{4}\right)\right]$$

$$= \frac{17}{2}[40-12]$$

$= 17 \times 14 = 238$

16. Let $\alpha, \beta \in \mathbb{R}$ be such that the function

$$f(x) = \begin{cases} 2\alpha(x^2-2)+2\beta x & ,x<1 \\ (\alpha+3)x+(\alpha-\beta) & ,x\ge 1 \end{cases}$$

be differentiable at all $x \in \mathbb{R}$. Then $34(\alpha + \beta)$ is equal to

(1) 84 (2) 48

(3) 36 (4) 24

Ans. **(2)**

Sol. $$f(x) = \begin{cases} 2\alpha x^2 + 2\beta x - 4\alpha & ;\ x<1 \\ (\alpha+3)x+\alpha-\beta & ;\ x\ge 1 \end{cases}$$

$f(1^+) = 2\alpha - \beta + 3$, $f(1^-) = -2\alpha + 2\beta$

$2\alpha - \beta + 3 = 2\beta - 2\alpha \Rightarrow 4\alpha - 3\beta + 3 = 0$ ...(1)

$f'(1^+) = 4\alpha + 2\beta$, $f'(1^-) = \alpha + 3$

$4\alpha + 2\beta = \alpha + 3 \Rightarrow 3\alpha + 2\beta - 3 = 0$ ...(2)

Solving (1) & (2)

We get $\alpha = \frac{3}{17}$, $\beta = \frac{21}{17}$

$$\Rightarrow 34(\alpha+\beta) = 34 \times \frac{27}{17} = 48$$

17. From a lot containing 10 defective and 90 non-defective bulbs, 8 bulbs are selected one by one with replacement. Then the probability of getting at least 7 defective bulbs is :-

(1) $\frac{7}{10^7}$ (2) $\frac{81}{10^8}$

(3) $\frac{67}{10^8}$ (4) $\frac{73}{10^8}$

Ans. **(4)**

Sol. 10 defective & 90 non-defective

Req. probability = (7 def 1 fair) or (8 defective)

$$\text{Req. probability} = \frac{(10^7 \times 90) \times 8 + 10^8}{100^8}$$

$$= \frac{72 \times 10^8 + 10^8}{100^8} = \frac{73}{10^8}$$

18. The mean and variance of a data of 10 observations are 10 and 2, respectively. If an observations α in this data is replaced by β, then the mean and variance become 10.1 and 1.99, respectively. Then $\alpha + \beta$ equals.

(1) 10 (2) 15

(3) 5 (4) 20

Ans. **(4)**

Sol. Let first 10 numbers are $x_1, x_2, \ldots\ldots x_9, \alpha$

$$\Rightarrow \alpha + \sum_{i=1}^{9} x_i = 100 \Rightarrow \sum_{i=1}^{9} x_i = 100 - \alpha$$

$$\text{Variance} = \left(\frac{\sum x_i^2}{n}\right) - \left(\frac{\sum x_i}{n}\right)^2$$

$$\Rightarrow \frac{\sum x_i^2}{n} = 98$$

$$\Rightarrow x_1^2 + x_2^2 + \ldots\ldots x_9^2 + \alpha^2 = 1020 \Rightarrow \sum x_i^2 = 1020 - \alpha^2$$

In second case, let number are

$x_1, x_2, \ldots\ldots x_9, \beta$

$100 - \alpha + \beta = 101 \quad \alpha - \beta + 1 = 0$

$$\frac{\sum x_i^2 + \beta^2}{10} - (10.1)^2 = 1.99$$

$\beta^2 - \alpha^2 = 20$

$$\alpha = \frac{19}{2}$$

$$\beta = \frac{21}{2}$$

$$\alpha + \beta = \frac{19+21}{2} = 20$$

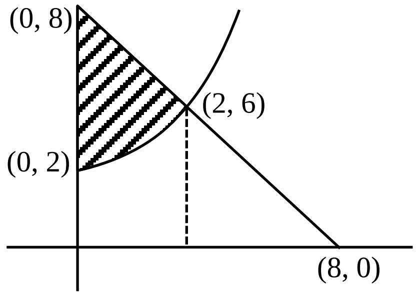
19. Let A_1 be the bounded area enclosed by the curves $y = x^2 + 2$, $x + y = 8$ and y-axis that lies in the first quadrant. Let A_2 be the bounded area enclosed by the curves $y = x^2 + 2$, $y^2 = x$, $x = 2$, and y-axis that lies in the first quadrant. Then $A_1 - A_2$ is equal to

(1) $\frac{2}{3}\left(2\sqrt{2}+1\right)$ (2) $\frac{2}{3}\left(4\sqrt{2}+1\right)$

(3) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (4) $\frac{2}{3}\left(3\sqrt{2}+1\right)$

Ans. (1)

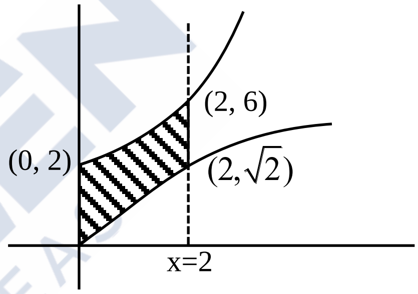
Sol.



$$A_1 = \int_0^2 ((8-x)-(x^2+2))\,dx$$

$$= A_1 = \int_0^2 (6-x-x^2)\,dx$$

$$A_1\left(6x - \frac{x^2}{2} - \frac{x^3}{3}\right)_0^2 = 12 - 2 - \frac{8}{3} = 10 - \frac{8}{3} = \frac{22}{3}$$



$$A_2 = \int_0^2 (x^2+2)dx - \frac{2}{3}(2\sqrt{2})$$

$$A_2 = \left(\frac{x^3}{3} + 2x\right)_0^2 - \frac{4\sqrt{2}}{3}$$

$$A_2 = \frac{8}{3} + 4 - \frac{4\sqrt{2}}{3} = \frac{20}{3} - \frac{4\sqrt{2}}{3}$$

$$A_1 - A_2 = \frac{2}{3} + \frac{4\sqrt{2}}{3}$$

20. The number of the real solutions of the equation :

$x|x+3| + |x-1| - 2 = 0$ is

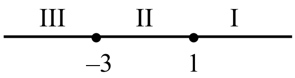
(1) 3 (2) 2

(3) 5 (4) 4

Ans. (1)

ALLEN OVERSEAS

Sol.



(I)

$x^2 + 3x + x - 1 - 2 = 0$

$x^2 + 4x - 3 = 0$

$x = -2 + \sqrt{7}$ (rejected) , $-2 - \sqrt{7}$ (rejected)

(II)

$x^2 + 3x + 1 - x - 2 = 0$

$x^2 + 2x - 1 = 0$

$x = -1 + \sqrt{2}, -1 - \sqrt{2}$

(III)

$-x^2 - 3x + 1 - x - 2 = 0$

$x^2 + 4x + 1 = 0$

$x = -2 - \sqrt{3}, -2 + \sqrt{3}$ (rejected)

SECTION-B

21. Let a differentiable function f satisfy the equation

$$\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x).$$

If $y = f(x)$ is a standard parabola passing through the points (2, 1) and (–4, β),

Then β^α is equal to _____.

Ans. (64)

Sol. $\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$, Put $\frac{tx}{36} = y$

$$\frac{dy}{dt} = \frac{x}{36}$$

$$\int_0^x \frac{f(y)36dy}{x} = 4\alpha f(x)$$

$$\int_0^x f(y)dy = \frac{\alpha f(x)x}{9}$$

$$f(x) = \frac{\alpha}{9}\left(f(x) + xf'(x)\right)$$

$$\left(1 - \frac{\alpha}{9}\right)f(x) = \frac{\alpha x}{9}f'(x) \Rightarrow (9-\alpha)f(x) = \alpha x f'(x)$$

$$\frac{f'(x)}{f(x)} = \left(\frac{9}{\alpha} - 1\right)\frac{1}{x}$$

$$\log_e f(x) = \left(\frac{9}{\alpha} - 1\right)\log_e x + \log_e c$$

$f(x) = cx^{\left(\frac{9}{\alpha}-1\right)}$ for standard parabola

$$\frac{9}{\alpha} - 1 = 2$$

$\alpha = 3$

$f(x) = cx^2$

passing through (2,1)

$1 = 4c \Rightarrow c = 1/4$

$y = \frac{x^2}{4}$ passing through (–4, β)

$\beta = 4$

$\beta^\alpha = 4^3 = 64$

22. Let a line L passing through the point P(1, 1, 1) be perpendicular to the lines $\frac{x-4}{4} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-17}{1} = \frac{y-71}{1} = \frac{z}{0}$. Let the line L intersect the yz-plane at the point Q. Another line parallel to L and passing through the point S(1,0, –1) intersects the yz-plane at the point R. Then the square of the area of the parallelogram PQRS is equal to _____.

Ans. (6)

Sol. $d_1 = \langle 4, 1, 1\rangle$ and $d_2 = \langle 1, 1, 0\rangle$

$$d_L = d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 4 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \langle -1, 1, 3\rangle$$

Line L passes through $P\langle 1, 1, 1\rangle$ with $d_2 = \langle -1, 1, 3\rangle$

$r(t) = \langle 1, 1, 1\rangle + t\langle -1, 1, 3\rangle$

$= \langle 1 - t, 1 + t, 1 + 3t\rangle$

$= \langle 1 - t, 1 + t, 1 + 3t\rangle$

For point Q; $x = 0$

$\Rightarrow t = 1$

$Q = \langle 0, 2, 4\rangle$

Another

Line parallel to L passes through $S \langle 1, 0, -1\rangle$

with $d_L = \langle -1, 1, 3\rangle$

$r'(u) = \langle 1, 0, -1\rangle + \mu\langle -1, 1, 3\rangle$

$= \langle 1 - \mu, \mu, -1 + 3\mu\rangle$

for point R, $x = 0$

$\Rightarrow \mu = 1$

$R\langle 0, 1, 2\rangle$

Area of parallelogram with adjacent vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

$\overrightarrow{PQ} = \langle -1, 1, 3\rangle$

$\overrightarrow{PS} = \langle 0, -1, -2\rangle$

Area of parallelogram

$$\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{vmatrix} i & j & k \\ -1 & 1 & 3 \\ 0 & -1 & -2 \end{vmatrix} = \langle 1, -2, 1\rangle$$

$$\text{Area} = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$$

23. The number of numbers greater than 5000, less than 9000 and divisible by 3, that can be formed using the digits 0, 1, 2, 5, 9, if the repetition of the digits is allowed, is ________

Ans. **(42)**

Sol. (1) all different

5, 0, 1, 9 $\Rightarrow \lfloor 3 = 6$ ways

(2) 2 alike, 2 different

5, 0, 0, 1 $\Rightarrow$ 3 ways

5, 1, 1, 2 $\Rightarrow$ 3 ways

5, 2, 2, 0 $\Rightarrow$ 3 ways

5, 2, 2, 9 $\Rightarrow$ 3 ways

5, 5, 0, 2 $\Rightarrow$ 6 ways

5, 5, 2, 9 $\Rightarrow$ 6 ways

5, 1, 9, 9 $\Rightarrow$ 3 ways

(3) 3 alike, 1 different

5, 5, 5, 0 $\Rightarrow$ 3 ways

5, 5, 5, 9 $\Rightarrow$ 3 ways

(4) 2 alike, 2 other alike

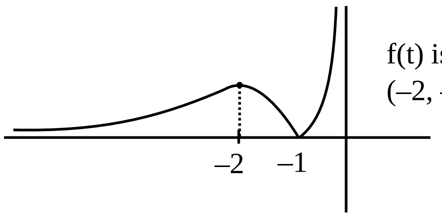
5, 5, 1, 1 $\Rightarrow$ 3 ways

Total ways = 42

24. Let $(2\alpha, \alpha)$ be the largest interval in which the function $f(t) = \dfrac{|t+1|}{t^2}$, $t < 0$, is strictly decreasing. Then the local maximum value of the function $g(x) = 2\log_e(x-2) + \alpha x^2 + 4x - \alpha$, $x > 2$, is ____

Ans. **(4)**

Sol. Drawing graph of f(t) for $t < 0$



f(t) is decreasing in $(-2, -1) \Rightarrow \alpha = -1$

$g(x) = \log_e(x-2) - x^2 + 4x + 1;\quad x > 2$

$$g'(x) = \frac{2}{x-2} - (2(x-2);\quad x > 2$$

$$g'(x) = \frac{1-(x-2)^2}{(x-2)} = \frac{-(x-3)(x-1)}{(x-2)}$$

$+$ $-$ as $x > 2$

2 3

maxima occur at $x = 3$

$g(3) = 2\log_e 1 - 9 + 12 + 1 = 4$

25. The number of 3×2 matrices A, which can be formed using the elements of the set $\{-2, -1, 0, 1, 2\}$ such that the sum of all the diagonal elements of A^TA is 5, is _____

Ans. (312)

Sol. $$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times 2}$$

$$A^TA = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}_{2\times 3} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times 2}$$

$$= \begin{pmatrix} a_1^2 + a_2^2 + a_3^2 & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & b_1^2 + b_2^2 + b_3^2 \end{pmatrix}$$

$\text{Tr}\,(A^TA) = a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 = 5$

$\{2, 1, 0, 0, 0, 0\}$

$\{2, -1, 0, 0, 0, 0\}$

$\{-2, 1, 0, 0, 0, 0\}$

$\{-2, -1, 0, 0, 0, 0\}$

$\{1, 1, 1, 1, 1, 0\}$

$$\text{No. of ways} = \frac{6!}{4!} \times 4 + 2 \times \frac{6!}{5!} + 2 \times \frac{6!}{4!} + 2 \times \frac{6!}{3!2!}$$

$$= \frac{6!}{3!} + 2 \times 6 + 2 \times 15 \times 2 \times \frac{6!}{3!}$$

$= 120 + 120 + 12 + 60 = 312$