# JEE–MAIN EXAMINATION – JANUARY 2026

**(HELD ON WEDNESDAY 28th JANUARY 2026)** **TIME : 3:00 PM TO 6:00 PM**

## MATHEMATICS — TEST PAPER WITH SOLUTION

### SECTION-A

**1.** Given below two statements :

**Statement I** : $25^{13} + 20^{13} + 8^{13} + 3^{13}$ is divisible by 7.

**Statement II** : The integral part of $\left(7+4\sqrt{3}\right)^{25}$ is an odd number.

In the light of the above statements, choose the **correct answer** from the options given below :

(1) Both **Statement I** and **Statement II** are false.

(2) Both **Statement I** and **Statement II** are true.

(3) **Statement I** is false but **Statement II** is true.

(4) **Statement I** is true but **Statement II** is false.

**Ans. (2)**

**Sol.** Statement I : $\underbrace{25^{13} + 3^{13}}_{\text{divisible by }(25+3)} + \underbrace{20^{13} + 8^{13}}_{\text{divisible by }(20+8)}$

$\therefore$ divisible by 7

**Statement II :** $R=\left(7+4\sqrt{3}\right)^{25}=I+f$

$R'=\left(7-4\sqrt{3}\right)^{25}=f'$

$\therefore R+R'=2\left[{}^{25}C_0 7^{25}+{}^{25}C_2 7^{23}\left(4\sqrt{3}\right)^2+....\right]$

I + f + f' = even integer

$\therefore$ I = odd integer

$\because\ 0<f+f'<2 \Rightarrow f+f'=1$

$\Rightarrow$ Both the statements are correct

**2.** The sum of the coefficients of $x^{499}$ and $x^{500}$ in $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+.......+x^{1000}$ is

(1) ${}^{1001}C_{501}$ (2) ${}^{1002}C_{500}$

(3) ${}^{1002}C_{501}$ (4) ${}^{1000}C_{501}$

**Ans. (2)**

**Sol.** $S=(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+....+x^{1000}$

$$=(1+x)^{1000}\frac{\left(1-\left(\frac{x}{1+x}\right)^{1001}\right)}{1-\frac{x}{1+x}}$$

$=(1+x)^{1001}-x^{1001}$

Required sum = ${}^{1001}C_{499}+{}^{1001}C_{500}={}^{1002}C_{500}$

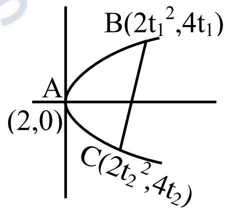
**3.** Let A be the focus of the parabola $y^2 = 8x$. Let the line $y = mx + c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $\left(\frac{7}{3},\frac{4}{3}\right)$, then $(BC)^2$ is equal to :

(1) 41 (2) 80

(3) 89 (4) 32

**Ans. (2)**

**Sol.**




Coordinates of centroid of triangle ABC are

$\frac{2}{3}(t_1^2+t_2^2+1)=\frac{7}{3} \Rightarrow t_1^2+t_2^2=\frac{5}{2}$

$\frac{4}{3}(t_1+t_2)=\frac{4}{3} \Rightarrow t_1+t_2=1$

$\left(t_1+t_2\right)^2=t_1^2+t_2^2+2t_1t_2 \Rightarrow t_1t_2=\frac{-3}{4}$

$(t_1-t_2)^2=(t_1+t_2)^2-4t_1t_2=4$

$(BC)^2=4\left(t_1^2-t_2^2\right)^2+16\left(t_1-t_2\right)^2$

$\Rightarrow (BC)^2 = 80$

**4.** The probability distribution of a random variable X is given below :

| X | 4k | $\frac{30}{7}k$ | $\frac{32}{7}k$ | $\frac{34}{7}k$ | $\frac{36}{7}k$ | $\frac{38}{7}k$ | $\frac{40}{7}k$ | 6k |
|---|---|---|---|---|---|---|---|---|
| P(X) | $\frac{2}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{1}{15}$ |

If $E(X) = \frac{263}{15}$, then $P(X < 20)$ is equal to :

(1) $\frac{3}{5}$ (2) $\frac{8}{15}$

(3) $\frac{11}{15}$ (4) $\frac{14}{15}$

**Ans. (3)**

**Sol.** $E(X) = \sum X_i P(X_i) = \frac{526k}{15\times 7} = \frac{263}{15} \Rightarrow k = \frac{7}{2}$

| X | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|
| P(X) | $\frac{2}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{1}{15}$ |

$P(X < 20) = \sum_{X=14}^{19} P(X) = \frac{11}{15}$

**5.** Let $f(x) = \lim_{\theta\to 0}\left(\frac{\cos \pi x - x^{\left(\frac{2}{\theta}\right)}\sin(x-1)}{1+x^{\left(\frac{2}{\theta}\right)}(x-1)}\right), x \in R$.

Consider the following two statements :

(I) f(x) is discontinous at x = 1.

(II) f(x) is continous at x = - 1.

Then,

(1) Neither (I) nor (II) is True

(2) Both (I) and (II) are True

(3) Only (II) is True

(4) Only (I) is True

**Ans. (1)**

**Sol.** $f(x) = \begin{cases} \cos \pi x & x \to 1^- \\ \frac{-\sin(x-1)}{(x-1)} & x \to 1^+ \end{cases}$

$RHL = \lim_{x\to 1}\frac{-\sin(x-1)}{(x-1)} = -1$

$LHL = \lim_{x\to 1}\cos \pi x = -1$, $f(1) = -1$

f(x) is continuous at x = 1

$f(x) = \begin{cases} \frac{-\sin(x-1)}{-(x-1)} & x \to -1^- \\ \cos \pi x & x \to -1^+ \end{cases}$

$RHL = \lim_{x\to -1}\cos \pi x = -1$

$LHL = \lim_{x\to -1}\frac{-\sin(x-1)}{(x-1)} = \frac{\sin 2}{-2}$

f(x) is discontinuous at x = –1

**6.** Considering the principal values of inverse trigonometric functions, the value of the expression $\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$ is equal to :

(1) $-\frac{33}{56}$ (2) $\frac{33}{56}$

(3) $\frac{16}{63}$ (4) $-\frac{16}{63}$

**Ans. (2)**

**Sol.** Let $\sin^{-1}\frac{2}{\sqrt{13}} = \theta$ & $\cos^{-1}\frac{3}{\sqrt{10}} = \phi$

$\sin\theta = \frac{2}{\sqrt{13}}$ & $\cos\phi = \frac{3}{\sqrt{10}}$

$\tan(2\theta - 2\phi) = \frac{\tan 2\theta - \tan 2\phi}{1 + \tan 2\theta \tan 2\phi}$

$\left(\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= \frac{\frac{12}{5} - \frac{3}{4}}{1 + \frac{12}{5}.\frac{3}{4}}$

$= \frac{33}{56}$

**7.** Let the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ be $\frac{5}{16}$, $a > 2$. If $\alpha$ is such that a, 4, $\alpha$, b are in A.P., then the equation $\alpha x^2 - ax + 2(\alpha - 2b) = 0$ has :

(1) One root in (1,4) and another in (–2,0)

(2) One root in (0,2) and another in (– 4, –2)

(3) Complex roots of magnitude less than 2

(4) Both roots in the interval (–2, 0)

**Ans.** **(1)**

**Sol.** $a = 4 - d,\ \alpha = 4 + d,\ b = 4 + 2d$

$\Rightarrow (4 + d)x^2 - (4 - d)x + 2(4 + d - 8 - 4d) = 0$

$\Rightarrow (4 + d)x^2 - (4 - d)x + 2(-4 - 3d) = 0$

Also $\dfrac{\frac{1}{a}+\frac{1}{b}}{2} = \dfrac{5}{16}$

$\Rightarrow \dfrac{\frac{1}{4-d}+\frac{1}{4+2d}}{2} = \dfrac{5}{16}$

$\Rightarrow d = 2$

Equation becomes

$6x^2 - 2x - 20 = 0$

$3x^2 - x - 10 = 0$

$x = 2, \dfrac{-5}{3}$

**8.** Given below are two statements :

**Statement I :** The function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = \dfrac{x}{1+|x|}$ is one-one.

**Statement II :** The function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = \dfrac{x^2+4x-30}{x^2-8x+18}$ is many-one.

In the light of the above statements, choose the **correct answer** from the options given below :

(1) Both **Statement I** and **Statement II** are false.

(2) Both **Statement I** and **Statement II** are true.

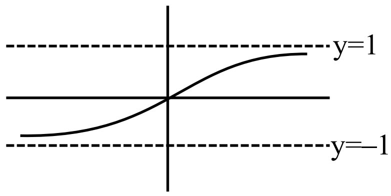
(3) **Statement I** is false but **Statement II** is true .

(4) **Statement I** is true but **Statement II** is false.

**Ans.** **(2)**

**Sol.** **Statement 1:** $f(x) = \dfrac{x}{1+|x|}$

$$f(x) = \begin{cases} \dfrac{x}{1+x} & x \ge 0 \\ \dfrac{x}{1-x} & x < 0 \end{cases}$$




f(x) is one-one

**Statement 2:** $f(x) = \dfrac{x^2+4x-30}{x^2-8x+18}$, $f(0) = \dfrac{-30}{18} = \dfrac{-5}{3}$

$\dfrac{-5}{3} = \dfrac{x^2+4x-30}{x^2-8x+18}$

On solving $x = 0, -1$

$\Rightarrow f(0) = f(-1) = \dfrac{-5}{3}$

$\therefore$ f(x) is many-one

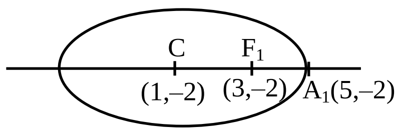
**9.** An ellipse has its center at (1,-2), one focus at (3,-2) and one vertex at (5, - 2). Then the length of its latus rectum is :

(1) $\dfrac{16}{\sqrt{3}}$ (2) 6

(3) $4\sqrt{3}$ (4) $6\sqrt{3}$

**Ans.** **(2)**

**Sol.**




$CA_1 = a = 4$

$CF_1 = ae = 2$

$e = \dfrac{1}{2}$

$LR = 2e\left(\dfrac{a}{e} - ae\right)$

$= 2 \times \dfrac{1}{2} \times \left(\dfrac{4}{1/2} - 2\right)$

$= 6$

**10.** Let the ellipse $E:\frac{x^2}{144}+\frac{y^2}{169}=1$ and the hyperbola $H:\frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$ have the same foci. If e and L respectively denote the eccentricity and the length of the latus rectum of H, then the value of 24(e + L) is :

(1) 296 (2) 126
(3) 148 (4) 67

**Ans. (1)**

**Sol.** Equation of hyperbola : $\frac{y^2}{\lambda^2}-\frac{x^2}{16}=1$

Equation of ellipse : $\frac{x^2}{144}+\frac{y^2}{169}=1$

$e' = \sqrt{1-\frac{144}{169}}=\frac{5}{13}$

focus $\Rightarrow$ (0, 5)

$\Rightarrow \lambda\sqrt{1+\frac{16}{\lambda^2}}=5$

$\Rightarrow \lambda^2 + 16 = 25$

$\lambda = 3$

Eccentricity of hyperbola $=\sqrt{1+\frac{16}{\lambda^2}}=\frac{5}{3}$

Length of latus rectum of hyperbola $=\frac{2(16)}{3}=\frac{32}{3}$

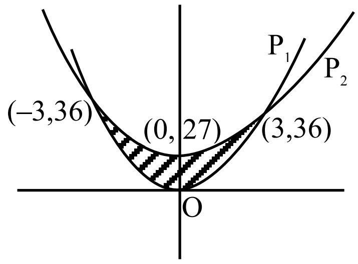
$24(e+\ell) = 24\left[\frac{5}{3}+\frac{32}{3}\right] = 8\times 37 = 296$

**11.** Let $P_1 : y = 4x^2$ and $P_2 : y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between $P_1$ and $P_2$ is six times the area of the bounded region enclosed between the line $y = \alpha x$, $\alpha > 0$ and $P_1$, then $\alpha$ is equal to :

(1) 8 (2) 15
(3) 12 (4) 6

**Ans. (3)**

**Sol.**




Area bounded between $P_1$ & $P_2$ is

$$\int_{-3}^{3}\left((x^2+27)-(4x^2)\right)dx$$

(P.O.I. of $P_1$ & $P_2$ is $x = \pm 3$)

$$=2\int_0^3(27-3x^2)dx = 2\left[27x-x^3\right]_0^3$$

$= 2[81 - 27] = 108$

$\therefore$ Area bounded between $P_1$ & L is 18 sq. units

(Area between $x^2 = 4ay$ & line $x = my$) is $\frac{8a^2}{3m^3}$

$\therefore$ Area between $x^2=\frac{y}{4}$ & $x=\frac{y}{\alpha}$ is

$$\frac{8.\left(\frac{1}{16}\right)^2}{3.\left(\frac{1}{\alpha}\right)^3}=18$$

$$\Rightarrow \frac{\frac{8}{16.16}}{\frac{3}{\alpha^3}}=18 \Rightarrow \alpha^3 = 2^6.3^3$$

$\Rightarrow \alpha = 12$

**12.** Let the circle $x^2 + y^2 = 4$ intersect x-axis at the points A(a, 0), a > 0 and B(b,0). Let P(2 cosα, 2 sinα), $0 < \alpha < \frac{\pi}{2}$ and Q(2 cosβ, 2 sinβ) be two points such that $(\alpha - \beta) = \frac{\pi}{2}$. Then the point of intersection of AQ and BP lies on :

(1) $x^2 + y^2 - 4y - 4 = 0$
(2) $x^2 + y^2 - 4x - 4 = 0$
(3) $x^2 + y^2 - 4x - 4y = 0$
(4) $x^2 + y^2 - 4x - 4y - 4 = 0$

**Ans. (1)**

**Sol.** Let point of intersection R(h,k)

$$m_{BR} = m_{BP} \Rightarrow \frac{k}{h+2} = \frac{2\sin\alpha}{2\cos\alpha+2} \Rightarrow \frac{k}{h+2} = \tan\frac{\alpha}{2}$$

$$m_{AR} = m_{AQ} \Rightarrow \frac{k}{h-2} = \frac{2\sin\beta}{2\cos\beta-2} = \frac{\sin\beta}{\cos\beta-1} = -\cot\frac{\beta}{2}$$

$$\frac{\alpha}{2} - \frac{\beta}{2} = \frac{\pi}{4}$$

$$\tan\left(\frac{\alpha}{2} - \frac{\beta}{2}\right) = \tan\frac{\pi}{4} = 1$$

$$\frac{\tan\frac{\alpha}{2} - \tan\frac{\beta}{2}}{1+\tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = 1$$

$$\frac{\frac{k}{h+2} + \frac{h-2}{k}}{1+\left(\frac{k}{h+2}\right)\left(\frac{2-h}{k}\right)} = 1 \Rightarrow \frac{\frac{k^2+h^2-4}{k(h+2)}}{\frac{4}{h+2}} = 1$$

$$\frac{h^2+k^2-4}{4k} = 1$$

$$x^2 + y^2 - 4y - 4 = 0$$

**13.** Let $[\cdot]$ denote the greatest integer function. Then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{12(3+[x])}{3+[\sin x]+[\cos x]}\right) dx$ is equal to:

(1) $15\pi + 4$ (2) $11\pi + 2$

(3) $13\pi + 1$ (4) $12\pi + 5$

**Ans.** **(2)**

**Sol.** $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{12(3+[x])dx}{3+[\sin x]+[\cos x]}$

$$I = \int_{-\frac{\pi}{2}}^{-1} \frac{12(1)dx}{2} + \int_{-1}^{0} \frac{12(2)dx}{2} + \int_{0}^{1} \frac{12(3)dx}{3} + \int_{1}^{\frac{\pi}{2}} \frac{12(4)dx}{3}$$

$$I = 6\left(\frac{\pi}{2}-1\right) + 12(0+1) + 12(1-0) + 16\left(\frac{\pi}{2}-1\right)$$

$$I = 3\pi - 6 + 12 + 12 + 8\pi - 16$$

$$I = 11\pi + 2$$

**14.** Let $y = y(x)$ be the solution of the differential equation $x\frac{dy}{dx} - y = x^2 \cot x, x \in (0,\pi)$.

If $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$ is equal to :

(1) $3\pi$ (2) $-3\pi$

(3) $-\pi$ (4) $\pi$

**Ans.** **(3)**

**Sol.** $xdy - ydx = x^2 \cot x\, dx$

$$x^2 d\left(\frac{y}{x}\right) = x^2 \cot x\, dx$$

$$d\left(\frac{y}{x}\right) = \cot x\, dx$$

$$\int d\left(\frac{y}{x}\right) = \int \cot x\, dx$$

$$\frac{y}{x} = \log_e \sin x + C$$

given $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

$\Rightarrow c = 1$

$y = x(\log_e \sin x + 1)$

$$y\left(\frac{\pi}{6}\right) = \frac{\pi}{6}[-\log_e 2 + 1]$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}[-\frac{1}{2}\log_e 2 + 1]$$

$$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$$

$$= \pi\left[\left(-\log_e 2 + 1\right) + 2\left(\frac{1}{2}\log_e 2 - 1\right)\right]$$

$$= \pi[1-2] = -\pi$$

**15.** The sum of all the elements in the range of f(x)=Sgn(sinx)+Sgn(cosx)+Sgn(tanx)+Sgn(cotx), $x \neq \frac{n\pi}{2}$, $n \in \mathbf{Z}$,

where $\text{Sgn}(t) = \begin{cases} 1, & \text{if } t > 0 \\ -1 & \text{if } t < 0 \end{cases}$, is

(1) 4 (2) 2

(3) –2 (4) 0

**Ans.** **(2)**

**Sol.** $x \in (0, \pi/2) \Rightarrow y = 1 + 1 + 1 + 1 = 4$

$x \in (\pi/2, \pi) \Rightarrow y = 1 - 1 - 1 - 1 = -2$

$x \in (\pi, 3\pi/2) \Rightarrow y = -1 - 1 + 1 + 1 = 0$

$x \in (3\pi/2, 2\pi) \Rightarrow y = -1 + 1 - 1 - 1 = -2$

$\therefore$ Range of y is $\{-2, 0, 4\}$

Required sum $= -2 + 0 + 4 = 2$

**16.** Let Q(a, b, c) be the image of the point P(3, 2, 1) in the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$. Then the distance of Q from the line $\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$ is

(1) 6

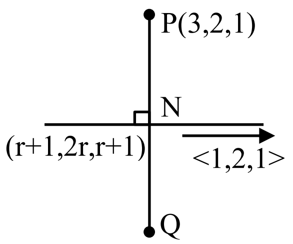
(2) 8

(3) 7

(4) 5

**Ans.** **(3)**

**Sol.**



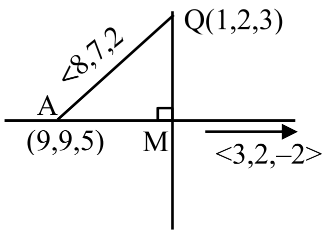

drs of PN = < r – 2, 2r – 2, r >

$1.(r-2) + 2(2r-2) + 1.(r) = 0$

$6r = 6 \Rightarrow r = 1$

$\therefore N \equiv (2, 2, 2)$

$\Rightarrow Q \equiv (1, 2, 3)$




$AQ = \sqrt{64+49+4} = \sqrt{117}$

$AM = \left|\frac{24+14-4}{\sqrt{9+4+4}}\right| = \frac{34}{\sqrt{17}} = 2\sqrt{17}$

$\therefore\ QM = \sqrt{117-68} = \sqrt{49} = 7$

**17.** Let P be a point in the plane of the vector $\overrightarrow{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{AC} = \hat{i} - \hat{j} + 3\hat{k}$ such that P is equidistant from the lines AB and AC. If $|\overrightarrow{AP}| = \frac{\sqrt{5}}{2}$, then the area of the triangle ABP is :

(1) 2

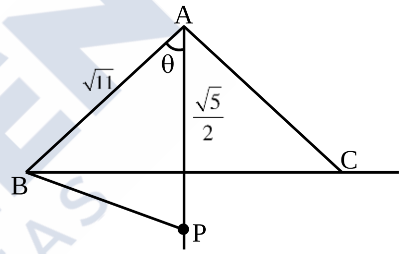
(2) $\frac{3}{2}$

(3) $\frac{\sqrt{30}}{4}$

(4) $\frac{\sqrt{26}}{4}$

**Ans.** **(3)**

**Sol.** $\cos 2\theta = \frac{3-1-3}{\sqrt{11}.\sqrt{11}} = -\frac{1}{11}$




$1 - 2\sin^2\theta = -\frac{1}{11} \Rightarrow 2\sin^2\theta = \frac{12}{11} \Rightarrow \sin\theta = \sqrt{\frac{6}{11}}$

$\therefore \text{Area}(\Delta APB) = \frac{1}{2} \times \sqrt{11} \cdot \frac{\sqrt{5}}{2} \cdot \sqrt{\frac{6}{11}} = \frac{\sqrt{30}}{4}$

**18.** Let

$A = \{z \in \mathbb{C} : |z-2| \le 4\}$ and

$B = \{z \in \mathbb{C} : |z-2| + |z+2| = 5\}$.

Then the max $\{|z_1 - z_2| : z_1 \in A \text{ and } z_2 \in B\}$ is

(1) $\frac{15}{2}$

(2) 8

(3) $\frac{17}{2}$

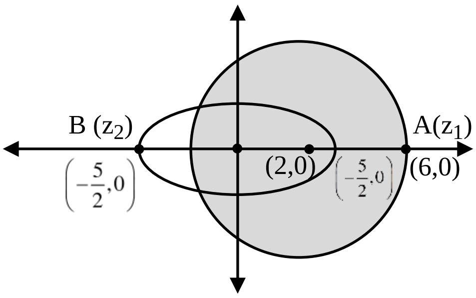
(4) 9

**Ans.** **(3)**

**Sol.** $|z-2| \le 4 \Rightarrow (x-2) + y^2 \le 16$

$$|z-2|+|z+2|=5 \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Rightarrow \frac{4x^2}{25}+\frac{4y^2}{9}=1$$




Maximum value of $|z_1-z_2| = 6+\frac{5}{2}=\frac{17}{2}$

**19.** Let $f(x)=\int \frac{dx}{x^{\left(\frac{2}{3}\right)}+2x^{\left(\frac{1}{2}\right)}}$ be such that $f(0) = -26 + 24 \log_e(2)$. If $f(1) = a + b \log_e(3)$, where $a, b \in \mathbf{Z}$, then $a + b$ is equal to:

(1) –18  (2) –5

(3) –11  (4) –26

**Ans.** **(3)**

**Sol.** $f(x)=\int \frac{dx}{x^{2/3}+2x^{1/2}}$

Put $x = t^6 \Rightarrow dx = 6t^5dt$

$$=\int \frac{6t^5dt}{t^4+2t^3}=6\int \frac{(t^2-4)+4}{t+2}dt$$

$$=6\left[\int (t-2)dt+4\int \frac{1}{t+2}dt\right]$$

$$=6\left[\frac{t^2}{2}-2t+4\ell n(t+2)\right]+C$$

$= 3x^{1/3} - 12x^{1/6} + 24\ell n(x^{1/6} + 2) + C$

$f(0) = 24\ell n2 + C = -26 + 24\ell n2$ (given)

$\Rightarrow C = -26$

Now

$f(1) = -35 + 24\ell n3 = a + b\ell n3$ (as given in ques.)

$\Rightarrow a = -35$ & $b = 24$

$\Rightarrow a + b = -11$

**20.** $\frac{6}{3^{26}}+\frac{10.1}{3^{25}}+\frac{10.2}{3^{24}}+\frac{10.2^2}{3^{23}}+\ldots+\frac{10.2^{24}}{3}$ is equal to

(1) $2^{25}$  (2) $2^{26}$

(3) $3^{25}$  (4) $3^{26}$

**Ans.** **(2)**

**Sol.** $S=\frac{6}{3^{26}}+\frac{10}{3^{25}}\left[\frac{(6)^{25}-1}{6-1}\right]$

$$S=\frac{6}{3^{26}}+\frac{10}{3^{25}}\left[\frac{6^{25}-1}{5}\right]$$

$$S=\frac{2}{3^{25}}+2\left[2^{25}-\frac{1}{3^{25}}\right]$$

$$S = 2^{26}$$

## SECTION-B

**21.** If $\sum_{r=1}^{25}\left(\frac{r}{r^4+r^2+1}\right)=\frac{p}{q}$, where p and q are positive integers such that gcd (p, q) = 1, then p + q is equal to ______.

**Ans.** **(976)**

**Sol.** $S=\sum \frac{r}{(r^2+r+1)(r^2-r+1)}$

$$=\frac{1}{2}\sum_{r=1}^{25}\left(\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right)$$

$$=\frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{601}-\frac{1}{651}\right)\right]$$

$$=\frac{1}{2}\left[\frac{1}{1}-\frac{1}{651}\right]$$

$$=\frac{1}{2}\left[\frac{650}{651}\right]=\frac{325}{651}$$

$$\frac{p}{q}=\frac{325}{651} \Rightarrow p+q=976$$

**22.** Three persons enter in a lift at the ground floor. The lift will go upto $10^{th}$ floor. The number of ways, in which the three persons can exit the lift at three different floors, if the lift does not stop at first, second and third floors, is equal to ______ .

**Ans.** **(210)**

**Sol.** ${}^7C_3 \times \underline{|3}$

$= 210$

**23.** Let f be a differentiable function satisfying

$$f(x) = 1 - 2x + \int_0^x e^{(x-t)}\, f(t)\,dt,\ x \in \mathbf{R} \text{ and let}$$

$$g(x) = \int_0^x (f(t)+2)^{15}\ (t-4)^6\ (t+12)^{17}\,dt,\ x \in \mathbf{R}.$$

If p and q are respectively the points of local minima and local maxima of g, then the value of |p + q| is equal to ________ .

**Ans. (9)**

**Sol.** $f(x) = 1 - 2x + e^x \int_0^x e^{-t} f(t)\,dt$

$$e^{-x} f(x) = (1-2x)e^{-x} + \int_0^x e^{-t} f(t)\,dt$$

$$e^{-x} f'(x) - e^{-x} f(x) = -2e^{-x} + (1-2x)e^{-x}(-1) + e^{-x} f(x)$$

$$f'(x) - 2f(x) = 2x - 3$$

$$\frac{dy}{dx} - 2y = 2x - 3$$

$$\Rightarrow y.e^{-2x} = \int e^{-2x}(2x-3)\,dx$$

On solving we get

$f(x) = 1 - x$

$$g(x) = \int_0^x (3-t)^{15}(t-4)^6(t+12)^{17}\,dt$$

$$g'(x) = (3-x)^{15}(x-4)^6(x+12)^{17}$$

$$= -(x-3)^{15}(x-4)^6(x+12)^{17}$$

$$\begin{array}{ccccccc} - & & + & & - & & - \\ & -12 & & 3 & & 4 & \end{array}$$

Local maxima $\Rightarrow q = 3$

Local minima $\Rightarrow p = -12 = |p + q| = 9$

**24.** If the distance of the point $P(43, \alpha, \beta)$, $\beta < 0$, from the line $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$, $\mu \in \mathbf{R}$ along a line with direction ratios 3, –1, 0 is $13\sqrt{10}$, then $\alpha^2 + \beta^2$ is equal to ________.

**Ans. (170)**

**Sol.** $\dfrac{x-43}{3} = \dfrac{y-\alpha}{-1} = \dfrac{z-\beta}{0} \Rightarrow P_1(43 + 3\lambda, \alpha - \lambda, \beta)$

$\dfrac{x-4}{2} = \dfrac{y}{0} = \dfrac{z+1}{3} \Rightarrow P_1(2\mu + 4, 0, 3\mu - 1)$

$\therefore\ \mu = \dfrac{3\lambda + 39}{2}$, $\alpha = \lambda$, $\beta = \dfrac{9\lambda - 115}{2}$

$P(43, \alpha, \beta)$, $P_1(43 + 3\alpha, 0, \beta)$

$(PP_1)^2 = 1690 = 10\alpha^2$, $\therefore\ \alpha = 13, \beta = 1$

$\therefore\ \alpha^2 + \beta^2 = 170$

**25.** Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B be two matrices such that $A^{100} = 100B + I$. Then the sum of all the elements of $B^{100}$ is ______.

**Ans. (0)**

**Sol.** $A = I + \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$, let $M = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

$$M^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = M^3 = M^4 = \ldots = M^{100}$$

$$A^{100} = (I + M)^{100} = \sum_{r=0}^{100} \binom{100}{r} M^r.I$$

$$A^{100} = I + 100M = I + 100B$$

$$\therefore\ M = B \Rightarrow M^{100} = B^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$