JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON WEDNESDAY 28th JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If $g(x) = 3x^2 + 2x - 3$, $f(0) = -3$ and $4g(f(x)) = 3x^2 - 32x + 72$, then $f(g(2))$ is equal to:

(1) $\frac{25}{6}$ (2) $-\frac{25}{6}$

(3) $\frac{7}{2}$ (4) $-\frac{7}{2}$

Ans. (3)

Sol. $g(2) = 13$

$f(g(2)) = f(13)$

Now $4g(f(x)) = 3x^2 - 32x + 72$

$4[3f^2(x) + 2f(x) - 3] = 3x^2 - 32x + 72$

Let $f(x) = t$

$12t^2 + 8t - (3x^2 - 32x + 84) = 0$

$$f(x) = \frac{-8 \pm \sqrt{64 + 48\left(3x^2 - 32x + 84\right)}}{24}$$

$$f(x) = \frac{-8 \pm 4(3x - 16)}{24}$$

$\because f(0) = -3 \quad \therefore$ we take +ve sign

$$\therefore f(x) = \frac{-8 + 4(3x - 16)}{24}$$

$$\therefore f(13) = \frac{-8 + 4 \times 23}{24} = \frac{84}{24} = \frac{7}{2}$$

2. The value of $\sum\limits_{k=1}^{\infty} (-1)^{k+1} \left(\frac{k(k+1)}{k!} \right)$ is :

(1) 2/e (2) 1/e

(3) $\sqrt{e}$ (4) e/2

Ans. (2)

Sol. $T_k = (-1)^{k+1} \cdot \frac{k(k+1)}{\underline{|k}} = (-1)^{k+1}\left(\frac{k(k-1) + 2k}{\underline{|k}}\right)$

$$\therefore \text{sum} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\underline{|k-2}} + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{\underline{|k-1}}$$

$$= \left(\frac{1}{\underline{|-1}} - \frac{1}{\underline{|0}} + \frac{1}{\underline{|1}} - \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} \ldots\right) + \left(\frac{2}{\underline{|0}} - \frac{2}{\underline{|1}} + \frac{2}{\underline{|2}} - \frac{2}{\underline{|3}} \ldots\right)$$

$$= \frac{1}{e}$$

3. Let $y = x$ be the equation of a chord of the circle C_1 (in the closed half-plane $x \geq 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2, which passes through the point (2, 3) and is farthest from the center of C_2, is $x + ay + b = 0$, then $a - b$ is equal to :

(1) 10 (2) –6

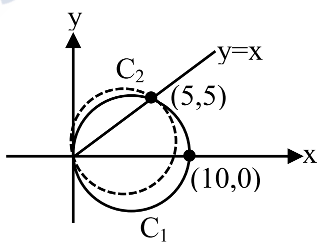
(3) –2 (4) 6

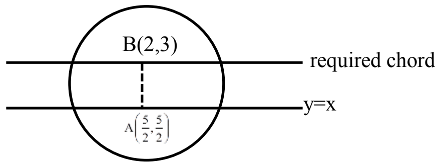
Ans. (3)

Sol. Equation of circle C_2 is

$x^2 + y^2 - 5x - 5y = 0$

its centre is $\left(\frac{5}{2}, \frac{5}{2}\right)$





$m_{AB} = -1$

$\therefore$ Slope of required chord = 1

$\therefore$ equation of required chord is $x - y + 1 = 0$

$\therefore a = -1, b = 2$

$\therefore a - b = -2$

4. If $\frac{\tan(A-B)}{\tan A}+\frac{\sin^2 C}{\sin^2 A}=1$, A, B, C $\in \left(0,\frac{\pi}{2}\right)$, then

(1) tan A, tan C, tan B are in G.P.

(2) tan A, tan B, tan C are in G.P.

(3) tan A, tan C, tan B are in A.P.

(4) tan A, tan B, tan C are in A.P.

Ans. **(1)**

Sol. $\frac{\tan A-\tan B}{(1+\tan A\tan B)\tan A}+\frac{1+\cot^2 A}{1+\cot^2 C}=1$

Put tanA = x, tanB = y, tanC = z

$\therefore \frac{x-y}{(1+xy)x}+\frac{(x^2+1)z^2}{x^2(z^2+1)}=1$

$\therefore x(x-y)(z^2+1)+z^2(1+x^2)(1+xy)$

$=(1+xy)x^2(1+z^2)$

after solving we get

$z^2 = xy$ $\quad\quad \because 1+x^2 \neq 0$

$\therefore \tan^2 C = \tan A.\tan B$

tanA, tanC, tanB are in G.P.

5. Let z be a complex number such that $|z-6|=5$ and $|z+2-6i|=5$.

Then the value of $z^3+3z^2-15z+141$ is equal to

(1) 42 (2) 37

(3) 50 (4) 61

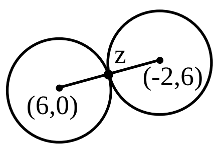
Ans. **(3)**

Sol. Center of first circle $C_1(6,0)$, $r_1=5$

Center of second circle $C_2(-2,6)$, $r_2=5$

$\because C_1C_2 = r_1 + r_2$

$\therefore$ common point Z is mid point of C_1 & C_2



$\therefore z = 2+3i$

$\therefore z^2 = 4z-13$

$\therefore z^3 = 3z-52$

$\therefore z^3+3z^2-15z+141 = 50$

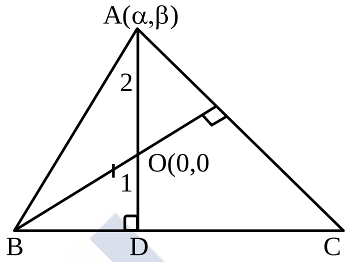
6. Let ABC be an equilateral triangle with orthocenter at the origin and the side BC on the line $x+2\sqrt{2}y=4$. If the co-ordinates of the vertex A are (α, β), then the greatest integer less than or equal to $|\alpha+\sqrt{2}\beta|$ is

(1) 2 (2) 3

(3) 5 (4) 4

Ans. **(4)**

Sol.



$\because m_{BC}.m_{AD}=-1$

$\Rightarrow \left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{\beta}{\alpha}\right)=-1$

$\Rightarrow \beta = 2\sqrt{2}\alpha$ ....(1)

$\because OD=\left|\frac{-4}{\sqrt{1+8}}\right|=\frac{4}{3}\Rightarrow AO=\frac{8}{3}$

So $AD=\frac{8}{3}+\frac{4}{3}=4$

$\Rightarrow \frac{|\alpha+2\sqrt{2}\beta-4|}{3}=4\Rightarrow \alpha=\frac{16}{9}\text{ or }-\frac{8}{9}$

$\left\{\begin{array}{l}\because A(\alpha,\beta)\ \&\ (0,0)\text{ lies on same}\\ \text{side of given line}\end{array}\right\}$

$\therefore (\alpha,\beta)=\left(\frac{16}{9},\frac{32\sqrt{2}}{9}\right)$; (Rejected)

so $(\alpha,\beta)=\left(-\frac{8}{9},\frac{-16\sqrt{2}}{9}\right)$

$=\left[\left|\alpha+\sqrt{2}\beta\right|\right]=\left[\left|\frac{-8-32}{9}\right|\right]=4$

7. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9} . Let x be the number of 9-digit numbers formed using the digits of the set S such that only one digit is repeated and it is repeated exactly twice. Let y be the number of 9-digit numbers formed using the digits of the set S such that only two digits are repeated and each of these is repeated exactly twice. Then,

(1) $29x = 5y$ (2) $45x = 7y$

(3) $21x = 4y$ (4) $56x = 9y$

Ans. **(3)**

Sol. $S = \{1, 2, 3, \ldots, 9\}$

$$x = {}^9C_1 \cdot {}^8C_7 \times \frac{9!}{2} = \frac{9\times 8\times 9!}{2}$$

$$y = {}^9C_2 \cdot {}^7C_5 \times \frac{9!}{2!\times 2!} = \frac{9\times 8}{2}\times\frac{7\times 6}{2}\times\frac{9!}{2!\times 2!}$$

$$\Rightarrow \frac{x}{y} = \frac{4}{21}$$

$21x = 4y$

8. Let $S = \{x^3 + ax^2 + bx + c : a, b, c \in N$ and $a, b, c \le 20\}$ be a set of polynomials. Then the number of polynomials in S, which are divisible by $x^2 + 2$, is

(1) 20 (2) 6

(3) 120 (4) 10

Ans. **(4)**

Sol. $$x^3 + ax^2 + bx + c = (x^2 + 2)\left(x + \frac{c}{2}\right)$$

$$x^2 : a = \frac{c}{2}$$

$x : b = 2$

$$b = 2, a = \frac{c}{2} \quad c \in \{2,4,\ldots,20\}$$

Number of polynomials in 'S' will be 10.

9. A bag contains 10 balls out of which k are red and (10 – k) are black, where $0 \le k \le 10$. If three balls are drawn at random without replacement and all of them are found to be black, then the probability that the bag contains 1 red and 9 black balls is :

(1) $\frac{7}{11}$ (2) $\frac{7}{55}$

(3) $\frac{7}{110}$ (4) $\frac{14}{55}$

Ans. **(4)**

Sol. $$\text{Probability} = \frac{{}^1C_0 \cdot {}^9C_3}{\sum_{k=0}^{10} {}^kC_0 \cdot {}^{10-k}C_3}$$

$$= \frac{{}^9C_3}{{}^{10}C_3 + {}^9C_3 + {}^8C_3 + \ldots + {}^3C_3}$$

$$= \frac{{}^9C_3}{{}^{11}C_4} = \frac{14}{55}$$

10. Ans. ( )

Sol. $$\frac{\beta-\alpha}{\alpha\beta} = \frac{1}{3}, \quad \alpha+\beta = \frac{\lambda+3}{\lambda}, \alpha\beta = \frac{3}{\lambda}$$

$$\beta - \alpha = \frac{\alpha\beta}{3} = \frac{1}{\lambda}$$

on squaring

$$\alpha^2 + \beta^2 - 2\alpha\beta = \frac{1}{\lambda^2} \quad \ldots\ldots\ldots(1)$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{(\lambda+3)^2}{\lambda^2} \quad \ldots\ldots\ldots\ldots (2)$$

(2) – (1) $$4\alpha\beta = \frac{(\lambda+3)^2 - 1}{\lambda^2}$$

$$\frac{12}{\lambda} = \frac{\lambda^2 + 6\lambda + 8}{\lambda^2}$$

$\Rightarrow \lambda^2 - 6\lambda^2 + 8\lambda = 0$

$\Rightarrow \lambda = 0,2,4$

Sum of possible values of λ is = 6

10. If α, β, where $\alpha < \beta$, are the roots of the equation $\lambda x^2 - (\lambda + 3)x + 3 = 0$ such that $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$, then the sum of all possible values of λ is :

(1) 6 (2) 2

(3) 4 (4) 8

Ans. (1)

Sol. $\frac{\beta-\alpha}{\alpha\beta} = \frac{1}{3}$, $\alpha+\beta = \frac{\lambda+3}{\lambda}, \alpha\beta = \frac{3}{\lambda}$

$$\beta - \alpha = \frac{\alpha\beta}{3} = \frac{1}{\lambda}$$

on squaring

$$\alpha^2 + \beta^2 - 2\alpha\beta = \frac{1}{\lambda^2} \quad \text{........(1)}$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{(\lambda+3)^2}{\lambda^2} \quad \text{.......... (2)}$$

(2) – (1) $\quad 4\alpha\beta = \frac{(\lambda+3)^2 - 1}{\lambda^2}$

$$\frac{12}{\lambda} = \frac{\lambda^2 + 6\lambda + 8}{\lambda^2}$$

$\Rightarrow \lambda^2 - 6\lambda^2 + 8\lambda = 0$

$\Rightarrow \lambda = 0, 2, 4$

Sum of possible values of λ is = 6

11. If $\int \left(\frac{1 - 5\cos^2 x}{\sin^5 x \cos^2 x}\right) dx = f(x) + C$ where C is the constant of integration, then $f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{1}{\sqrt{3}}\left(26 + \sqrt{3}\right)$ (2) $\frac{4}{\sqrt{3}}\left(8 - \sqrt{6}\right)$

(3) $\frac{1}{\sqrt{3}}\left(26 - \sqrt{3}\right)$ (4) $\frac{2}{\sqrt{3}}\left(4 + \sqrt{6}\right)$

Ans. (2)

Sol. $\int \frac{dx}{\sin^5 \cos^2 x} - 5\int \frac{dx}{\sin^5 x}$

$$\int \frac{\sec^2 x dx}{\sin^5 x} - 5\int \frac{dx}{\sin^5 x}$$

By IBP

$$= \frac{\tan x}{\sin^5 x} - \int -\frac{5}{\sin^6 x}.\cos x.\tan x dx - 5\int \frac{dx}{\sin^5 x}$$

$$= \frac{\tan x}{\sin^5 x} + c$$

$$f(x) = \frac{\tan x}{\sin^5 x}$$

$$f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right) = \frac{2^5}{\sqrt{3}} - \left(\sqrt{2}\right)^5 = 4\sqrt{2} - \frac{32}{\sqrt{3}}$$

$$= \frac{32}{\sqrt{3}} - 4\sqrt{2}$$

$$= \frac{4}{\sqrt{3}}\left(8 - \sqrt{6}\right)$$

12. Let f be a polynomial function such that $f(x^2 + 1) = x^4 + 5x^2 + 2$, for all $x \in \mathbb{R}$.

Then $\int_0^3 f(x)dx$ is equal to

(1) $\frac{41}{3}$ (2) $\frac{33}{2}$

(3) $\frac{27}{2}$ (4) $\frac{5}{3}$

Ans. (2)

Sol. $\because$ $f(x^2 + 1) = x^4 + 5x^2 + 2$

$\{$put $x^2 + 1 = t\}$

$\Rightarrow f(t) = (t - 1)^2 + 5(t - 1) + 2$

$\Rightarrow f(t) = t^2 + 3t - 2$

Now, $\int_0^3 f(t)dt = \int_0^3 (t^2 + 3t - 2)dt$

$$\left[\frac{t^3}{3} + \frac{3t^2}{2} - 2t\right]_0^3$$

$$\left[\frac{27}{3} + \frac{27}{2} - 6\right]$$

$$= \frac{33}{2}$$

13. The area of the region

$R=\{(x, y): xy \le 8,\ 1 \le y \le x^2,\ x \ge 0\}$ is

(1) $\frac{1}{3}\left(49\log_e(2)-15\right)$

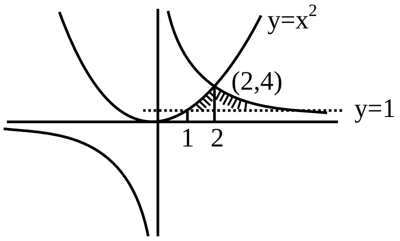
(2) $\frac{2}{3}\left(20\log_e(2)+9\right)$

(3) $\frac{2}{3}\left(24\log_e(2)-7\right)$

(4) $\frac{1}{3}\left(40\log_e(2)+27\right)$

Ans. (3)

Sol.



$$A=\int_1^2\left(x^2-1\right)dx+\int_2^8\left(\frac{8}{x}-1\right)dx$$

$$A = 8\log_e 4-\frac{14}{3} = 16\log_e 2-\frac{14}{3}$$

$$=\frac{2}{3}\left(24\log_e 2-7\right)$$

14. The value of

$$\lim_{x\to 0}\frac{\log_e\left(\sec(ex).\sec\left(e^2x\right)\cdot\ldots\cdot\sec\left(e^{10}x\right)\right)}{e^2-e^{2\cos x}}$$

is equal to

(1) $\frac{(e^{10}-1)}{2e^2(e^2-1)}$ (2) $\frac{(e^{20}-1)}{2e^2(e^2-1)}$

(3) $\frac{(e^{20}-1)}{2(e^2-1)}$ (4) $\frac{(e^{10}-1)}{2(e^2-1)}$

Ans. (3)

Sol. $\Rightarrow \lim\limits_{x\to 0}\dfrac{\ln(\sec(ex))+\ln(\sec(e^2x))+\ldots..\ell n(\sec(e^{10}x))}{e^{2\cos x}\left(\dfrac{e^{2-2\cos x}-1}{2-2\cos x}\right)\times\dfrac{2-2\cos x}{x^2}\times x^2}$

$$\Rightarrow \lim_{x\to 10}\frac{\ell n(\sec(ex))+\ell n(\sec(e^2x))+\ldots..\ell n(\sec(e^{10}x))}{e^2\ x^2}$$

Using L'H rule

$$\Rightarrow \lim_{x\to 10}\frac{e\tan ex+e^2\tan e^2x+\ldots..+e^{10}\tan e^{10}x}{2e^2x}$$

$$\Rightarrow \frac{1}{2e^2}\left[e^2+e^4+e^6+\ldots.+e^{20}\right]$$

$$\Rightarrow \frac{1}{2}\frac{e^2((e^2)^{10}-1)}{e^2(e^2-1)}$$

$$\Rightarrow \frac{1}{2}\frac{(e^{20}-1)}{(e^2-1)}$$

15. The mean and variance of 10 observations are 9 and 34.2, respectively. If 8 of these observations are 2, 3, 5, 10, 11, 13, 15, 21, then the mean deviation about the median of all the 10 observations is

(1) 5 (2) 4

(3) 6 (4) 7

Ans. (1)

Sol. $\dfrac{2+3+5+10+11+13+15+21+a+b}{10}=9$

$\dfrac{80+a+b}{10}=9 \Rightarrow a+b=10$ ….. (1)

$$\frac{\Sigma x_i^2}{10}-\left(\frac{\Sigma x_i}{10}\right)^2=34.2$$

$$\frac{2^2+3^2+5^2+10^2+11^2+13^2+15^2+21^2+a^2+b^2}{10}-(9)^2=34.2$$

$1094+a^2+b^2-810=342$

$a^2+b^2=58$ ….(2)

$a=7, b=3$

or $a=3, b=7$

Number $\to$ 2, 3, 5, 7, 10, 11, 13, 15, 21

Mean $=\dfrac{7+10}{2}=8.5$

$$\text{M.D}=\frac{6.5+5.5+5.5+3.5+1.5+1.5+2.5+4.5+6.5+12.5}{10}$$

$$=\frac{50}{5}$$

$=5$

16. Let A, B and C be three 2×2 matrices with real entries such that $B = (I + A)^{-1}$ and $A + C = I$. If $BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}$ and $CB\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$, then $x_1 + x_2$ is

(1) 2 (2) 0

(3) –2 (4) 4

Ans. **(2)**

Sol. $B = (I + A)^{-1}, A + C = I$

$\Rightarrow B(I + A) = (I + A)B = I$

$\Rightarrow B + BA = B + AB$

$\Rightarrow B + B(I - C) = B + (I - C)B$

$\Rightarrow 2B - BC = 2B - CB$

$\Rightarrow BC = CB$

$$\therefore\ CB\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ -6 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 32 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad \therefore\ x_1 + x_2 = 0$$

17. The common difference of the A.P.: $a_1, a_2, \ldots, a_m$ is 13 more than the common difference of the A.P.: $b_1, b_2, \ldots, b_n$. If $b_{31} = -277$, $b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to

(1) 21 (2) 24

(3) 19 (4) 16

Ans. **(3)**

Sol. Let common difference of A.P.'s are d_1 & d_2

$\therefore\ d_1 = 13 + d_2$

$b_1 + 30d_2 = -277$ ....(1)

$b_1 + 42d_2 = -385$ ....(2)

By (2) – (1)

$12d_2 = -108$

$d_2 = -9$

$\therefore\ \boxed{d_1 = 4}$

Now $a_{78} = 327$

$\Rightarrow a_1 + 77d_1 = 327$

$\Rightarrow a_1 + 308 = 327$

$a_1 = 19$

18. If the distances of the point (1, 2, a) from the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$ along the lines

$L_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b}$ and

$L_2 : \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c}$ are equal,

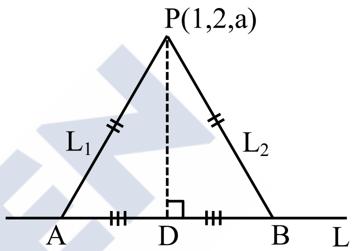
then a + b + c is equal to

(1) 7 (2) 5

(3) 6 (4) 4

Ans. **(1)**

Sol.



$$L : \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$$

$$L_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} = \lambda$$

$$L_2 : \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} = \mu$$

Let $A(3\lambda + 1, 4\lambda + 2, b\lambda + a)$

It lies on L

$$\therefore\ \frac{3\lambda}{1} = \frac{4\lambda + 2}{2} = \frac{b\lambda + a - 1}{1}$$

$\Rightarrow \boxed{\lambda = 1}$ and $a + b - 1 = 3$

$\Rightarrow A(4, 6, 4),\ a + b = 4$ ....(1)

Let $B(\mu + 1, 4\mu + 2, c\mu + a)$

It also lies on L

$$\frac{\mu}{1} = \frac{4\mu + 2}{2} = \frac{c\mu + a - 1}{1}$$

$\Rightarrow 2\mu = 4\mu + 2$

$\Rightarrow \boxed{\mu = -1}$

$a - c - 1 = -1$

$\Rightarrow \boxed{a = c}$ ....(2) & B(0, –2, 0) also PA = PB, P(1, 2, a), A(4, 6, 4)

$\Rightarrow 9 + 16 + (a-4)^2 = 1 + 16 + a^2$

$\Rightarrow 16 + 8 = 8a$

$\boxed{a=3}$ $\quad \therefore c = 3, b = 1$

$\therefore \boxed{a+b+c=7}$

19. For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying

$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$ and

$|2\vec{a} + k\vec{b} + k\vec{c}| = 3,$

the positive value of k is :

(1) 3 (2) 6

(3) 4 (4) 5

Ans. (4)

Sol. $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} - = -\frac{3}{2}$

$\Rightarrow \vec{a}+\vec{b}+\vec{c} = 0 \Rightarrow \vec{b}+\vec{c} = -\vec{a}$

$|2\vec{a} + k(\vec{b}+\vec{c})| = 3$

$|\vec{a}(2-k)| = 3$

K = 5 or – 1

Positive value of k is 5

20. Let y = y(x) be the solution of the differential equation

$$x\frac{dy}{dx} - \sin 2y = x^3(2-x^3)\cos^2 y, \ \ x \neq 0.$$

If y (2) = x, then tan(y(1)) is equal to

(1) $\frac{3}{4}$ (2) $\frac{7}{4}$

(3) $-\frac{7}{4}$ (4) $-\frac{3}{4}$

Ans. (2)

Sol. $x\frac{dy}{dx} - \sin 2y = x^3(2-x^3)\cos^2 y$

$\sec^2 y\frac{dy}{dx} - 2\tan y.\frac{1}{x} = x^2(2-x^3)$

$\tan y = t \Rightarrow \sec^2 y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2t}{x} = x^2(2-x^3)$ (LDE)

I.F. $= e^{\int -\frac{2}{x}dx} = e^{-2\ell n\, x} = \frac{1}{x^2}$

$\therefore \frac{t}{x^2} = \int \frac{1}{x^2}x^2(2-x^3)dx + C$

$\frac{\tan y}{x^2} = 2x - \frac{x^4}{4} + C$

$y(2) = 0 \Rightarrow 0 = 4 - 4 + C \Rightarrow C = 0$

$\tan y = 2x^3 - \frac{1}{4}x^6$

$x = 1 \Rightarrow \tan y = 2 - \frac{1}{4} = \frac{7}{4} \Rightarrow (2)$

SECTION-B

21. In a G.P., if the product of the first three terms is 27 and the set of all possible values for the sum of its first three terms is $\mathbb{R} - (a, b)$, then $a^2 + b^2$ is equal to ____.

Ans. (90)

Sol. Let first three terms of G.P. are $\frac{A}{r}, A, Ar$

$\frac{A}{r}.A.Ar = 27$

$A = 3$

$3\left(\frac{1}{r}+1+r\right) = 3 + 3\left(r+\frac{1}{r}\right)$

We know, $r+\frac{1}{r} \geq 2$ or $r+\frac{1}{r} \leq -2$

$S \in R - (-3, 9)$

$a^2 + b^2 = 9 + 81 = 90$

22. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2\sec^2\theta = 8$ be e_1 and ℓ_1, respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2\sec^2\theta + y^2 = 6$ be e_2 and ℓ_2, respectively. If $e_1^2 = e_2^2\left(\sec^2\theta + 1\right)$, then $\left(\frac{\ell_1\ell_2}{e_1e_2}\right)\tan^2\theta$ is equal to _____.

Ans. **(8)**

Sol. $\frac{x^2}{8} - \frac{y^2}{8\cos^2\theta} = 1$, $e_1 = \sqrt{1 + \frac{8\cos^2\theta}{8}}$

$\ell_1 = \frac{2b^2}{a} = \frac{2.(8\cos^2\theta)}{2\sqrt{2}}$

$\frac{x^6}{6} + \frac{y^2}{6\cos^2\theta} = 1$; $e_2 = \sqrt{1 - \frac{6\cos^2\theta}{6}} = \sin\theta$

$\ell_2 = \frac{2b^2}{a} = \frac{2.6\cos^2\theta}{\sqrt{6}}$

$e_1^2 = e_2^2\left(1 + \sec^2\theta\right)$

$1 + \cos^2\theta = \sin^2\theta\left(1 + \frac{1}{\cos^2\theta}\right)$

$1 + \cos^2\theta = \sin^2\theta + \tan^2\theta$

Solving we get $\theta = \frac{\pi}{4}$

$\ell_1 = 2\sqrt{2}$

$e_1 = \sqrt{\frac{3}{2}}$

$\ell_2 = \sqrt{6}$

$e_2 = \frac{1}{\sqrt{2}}$

$\left(\frac{\ell_1\ell_2}{e_1e_2}\right)\tan^2\theta = 8$ (By putting values)

23. If $k = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)\right)$ then the number of solutions of the equation $\sin^{-1}(kx - 1) = \sin^{-1}x - \cos^{-1}x$ is ______.

Ans. **(1)**

Sol. Let $\theta = \frac{1}{2}\sin^{-1}\frac{2}{3}$, then $\frac{1}{2}\cos^{-1}\frac{1}{3} = \left(\frac{\pi}{4} - \theta\right)$

$k = \tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$

$k = \frac{2}{\frac{2}{3}} = 3$

$\sin^{-1}(3x - 1) = \sin^{-1}x - \cos^{-1}x$

$\sin^{-1}(3x - 1) = \frac{\pi}{2} - 2\cos^{-1}x$

$3x - 1 = \sin\left(\frac{\pi}{2} - 2\cos^{-1}x\right)$

$3x - 1 = 2x^2 - 1 \Rightarrow x = 0, \frac{3}{2}$ (rejected)

No. of solution = 1

24. The value of $\sum_{r=1}^{20}\left(\left|\sqrt{\pi\left(\int_0^r x\,|\sin \pi x|\,dx\right)}\right|\right)$ is ____.

Ans. **(210)**

Sol. Let $I_r = \int_0^r x|\sin \pi x|dx$ ....(1)

Apply King Property

$= \int_0^r (r - x)|\sin \pi x|dx$ ....(2)

By (1) + (2)

$2I_r = \int_0^r r|\sin \pi x|dx \Rightarrow I_r = \frac{r}{2}\int_0^r |\sin \pi x|dx$

$I_1 = \frac{1}{2}\int_0^1 |\sin \pi x|dx = \frac{1}{2\pi}\int_0^{\pi}|\sin t|dt = \frac{1}{2\pi}(2)$

$$I_2 = \frac{2}{2}\int_0^2 |\sin \pi x|dx = \frac{2}{2\pi}\int_0^{2\pi}|\sin t|dt = \frac{2}{2\pi}(4)$$

$$S = \sqrt{\pi.\frac{1}{2\pi}.2} + \sqrt{\pi.\frac{2}{2\pi}.4} + \sqrt{\pi.\frac{3}{2\pi}.6} + ... + \sqrt{\pi.\frac{20}{2\pi}(2.20)}$$

$= 1 + 2 + 3 + ... + 20$

$$= \frac{20\times 21}{2} = 210$$

25. Let PQR be a triangle such that $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}$, a, b $\in \mathbb{Z}$. Let S be the point on QR, which is equidistant from the lines PQ and PR. If $|\overrightarrow{PR}| = 9$ and $\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of $3a - 4b$ is ___.

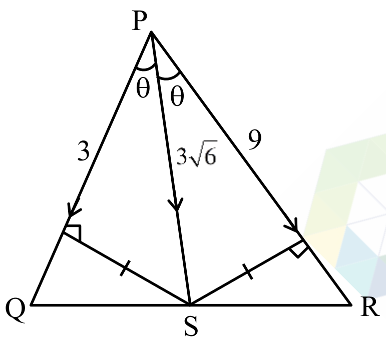
Allen Ans. (Bonus)

NTA Ans. (37)

Sol. $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}$ (a, b $\in$ Z)

$\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$



$\left|\overrightarrow{PR}\right| = 9$

$a^2 + b^2 + 16 = 81$

$a^2 + b^2 = 65$ ....(1)

$$\cos\theta = \frac{\overrightarrow{PQ}.\overrightarrow{PS}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PS}\right|}$$

$$= \frac{-2+7+4}{3.3\sqrt{6}} = \frac{\cancel{9}}{\cancel{3.3}\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}}$$

$$\frac{1}{\sqrt{6}} = \frac{\overrightarrow{PS}.\overrightarrow{PR}}{\left|\overrightarrow{PS}\right|\left|\overrightarrow{PR}\right|} = \frac{a-7b-8}{3\sqrt{6}.9}$$

$a - 7b = 35$ ....(2)

From (1) & (2)

$\Rightarrow a = 7, b = -4$

$\therefore 3a - 4b = 21 + 16 = 37$

(Matched with NTA)

But here PS is internal Angle bisector of $\angle QPR$ hence

$$PS = \frac{2PQ.PR}{PQ + PR}\cos\theta$$

$$\Rightarrow 3\sqrt{6} = \frac{2\times 3\times 9}{3+9}\cos\theta$$

$$\Rightarrow \cos\theta = \frac{2\sqrt{6}}{3} = \frac{2\sqrt{2}}{\sqrt{3}} > 1 \text{ (Not possible)}$$

Hence NO such triangle is possible

Another way to check

For evaluated value of a & b [a = 7, b = –4]

$\overrightarrow{PR} = 7\hat{i} - 4\hat{j} - 4\hat{k}$

$\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$

$\therefore \overrightarrow{QS} = \overrightarrow{PS} - \overrightarrow{PQ} = 3\hat{i} - 6\hat{j}$

$\overrightarrow{SR} = \overrightarrow{PR} - \overrightarrow{PS} = 6\hat{i} + 3\hat{j} - 6\hat{k}$

$\therefore \overrightarrow{QS}$ is not parallel to $\overrightarrow{SR}$

$\therefore$ Q,S,R are not co-linear

(Bonus)