JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON WEDNESDAY 2nd APRIL 2025) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The largest $n \in N$ such that 3^n divides 50! is:

(1) 21 (2) 22

(3) 20 (4) 23

Ans. (2)

Sol. $2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}$

$$B = \left[\frac{50}{3}\right] + \left[\frac{50}{3^2}\right] + \left[\frac{50}{3^3}\right] + \left[\frac{50}{3^4}\right]$$

$= 16 + 5 + 1$

$= 2$

Maximum value of n is 22

2. Let one focus of the hyperbola H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be at $(\sqrt{10}, 0)$ and the corresponding directrix be $x = \frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H, then $9(e^2 + l)$ is equal to:

(1) 14 (2) 15

(3) 16 (4) 12

Ans. (3)

Sol. $ae = \sqrt{10}$ and $\frac{a}{e} = \frac{9}{10}$

$\Rightarrow a^2 = 9$ and $e = \frac{\sqrt{10}}{3}$

Now $(ae)^2 = a^2 + b^2$

$10 = 9 + b^2 \Rightarrow b^2 = 1$

$\ell = \frac{2b^2}{a} = \frac{2(1)}{3}$

$\Rightarrow 9(e^2 + \ell)$

$= 9\left(\frac{10}{9} + \frac{2}{3}\right)$

$= 10 + 6$

$= 16$

3. The number of sequences of ten terms, whose terms are either 0 or 1 or 2, that contain exactly five 1s and exactly three 2s, is equal to

(1) 360 (2) 45

(3) 2520 (4) 1820

Ans. (3)

Sol. 11111 2 2 2 0 0

No. of sequences $= \frac{10!}{5!3!2!} = 2520$

Note : Sequence can start with 0.

4. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $(\sin x \cos y)(f(2x+2y) - f(2x-2y)) = (\cos x \sin y)(f(2x+2y) + f(2x-2y))$, for all x, y $\in \mathbf{R}$.

If $f'(0) = \frac{1}{2}$, then the value of $24 f''\left(\frac{5\pi}{3}\right)$ is:

(1) 2 (2) –3

(3) 3 (4) –2

Ans. (2)

Sol. $(\sin x \cos y)(f(2x + 2y) - f(2x - 2y)) = (\cos x \sin y)(f(2x + 2y) + f(2x - 2y))$

$f(2x + 2y)(\sin(x-y)) = f(2x - 2y)\sin(x + y)$

$$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)}$$

Put $2x + 2y = m$, $2x - 2y = n$

$$\frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \frac{f(n)}{\sin\left(\frac{n}{2}\right)} = K$$

$\Rightarrow f(m) = K\sin\left(\frac{m}{2}\right)$

$\therefore f(x) = K\sin\left(\frac{x}{2}\right)$

$f'(x) = \frac{K}{2}\cos\left(\frac{x}{2}\right)$

Put $x = 0$; $\frac{1}{2} = \frac{K}{2} \Rightarrow K = 1$

$f'(x) = \frac{1}{2}\cos\frac{x}{2}$

$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$

$4f''\left(\frac{5\pi}{3}\right) = \left(-\frac{1}{4}\sin\left(\frac{5\pi}{6}\right)\right)24$

$= \frac{-24}{8} = -3$

5. Let $A = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}$, $\alpha > 0$, such that det(A) = 0 and $\alpha + \beta = 1$. If I denotes 2×2 identity matrix, then the matrix $(1 + A)^8$ is:

(1) $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

(3) $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$ (4) $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Ans. (4)

Sol. $|A| = 0$

$\alpha\beta + 6 = 0$

$\alpha\beta = -6$

$\alpha + \beta = 1$

$\Rightarrow \alpha = 3, \beta = -2$

$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$

$\therefore A^2 = A$

$A = A^2 = A^3 = A^4 = A^5$

$(I + A)^8$

$= I + {}^8C_1A^7 + {}^8C_2A^6 + \ldots + {}^8C_8A^8$

$= I + A\,({}^8C_1 + {}^8C_2 + \ldots + {}^8C_8)$

$= I + A(2^8 - 1)$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 765 & -255 \\ 1530 & -510 \end{bmatrix}$

$= \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

6. The term independent of x in the expansion of $\left(\frac{(x+1)}{\left(x^{2/3}+1-x^{1/3}\right)} - \frac{(x+1)}{\left(x-x^{1/2}\right)}\right)^{10}$, $x > 1$ is:

(1) 210 (2) 150

(3) 240 (4) 120

Ans. (1)

Sol. $\left(\frac{(x+1)}{\left(x^{\frac{2}{3}}+1-x^{\frac{1}{3}}\right)} - \frac{(x-1)}{\left(x-x^{\frac{1}{2}}\right)}\right)^{10}$

$= \left(\left(x^{\frac{1}{3}}+1\right) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$= \left(x^{\frac{1}{3}} - \frac{1}{\sqrt{x}}\right)^{10}$

$T_{r+1} = {}^{10}C_r(x)^{\frac{10-r}{3}}(-1)^r(x)^{-\frac{r}{2}}$

$\frac{10-r}{3} - \frac{r}{2} = 0$

$(20 - 2r) - 3r = 0$

$r = 4$

$\Rightarrow {}^{10}C_4(-1)^4 = 210$

7. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta + \left(2-\sqrt{6}\right)\cos\theta - \sqrt{3} = 0$, is equal to:

(1) 12 (2) 6

(3) 8 (4) 10

Ans. (3)

Sol. $2\sqrt{2}\cos^2\theta + 2\cos\theta - \sqrt{6}\cos\theta - \sqrt{3} = 0$

$(2\cos\theta - \sqrt{3})\ (\sqrt{2}\cos\theta + 1) = 0$

$\cos\theta = \frac{\sqrt{3}}{2}, \frac{-1}{\sqrt{2}}$

Number of solution = 8

8. Let a_1, a_2, a_3.... be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1, a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is:

(1) 11 (2) 10

(3) 18 (4) 17

Ans. **(1)**

Sol. Let $a_1 = a$, common difference = d

$a_1 + a_3 + a_5 + \ldots\ldots + a_{23} = -\frac{72}{5}a$

$\frac{12}{2}[2a + 11 \times 2d] = -\frac{72}{5}a$

$12a + 132d = -\frac{72}{5}a$

$132a + 132 \times 5d = 0$

$a = -5d$

$\frac{n}{2}(2a + (n-1)d) = 0 \Rightarrow -10d + nd - d = 0$

$n = 11$

9. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local maximum and local minimum values at p and q, respectively, such that $p^2 = q$, then $f(3)$ is equal to:

(1) 55 (2) 10

(3) 23 (4) 37

Ans. **(4)**

Sol. $f'(x) = 6x^2 - 18ax + 12a^2$

$f'(x) = 6(x^2 - 3ax + 2a^2)$

roots are a, 2a

$p^2 = q \Rightarrow a^2 = 2a$

$a = 2$

$f(x) = 2x^3 - 18x^2 + 48x + 1$

$f(3) = 37$

10. Let z be a complex number such that $|z| = 1$. If $\frac{2+k^2z}{k+\bar{z}} = kz$, $k \in \mathbf{R}$, then the maximum distance of $k + ik^2$ from the circle $|z - (1 + 2i)| = 1$ is:

(1) $\sqrt{5}+1$ (2) 2

(3) 3 (4) $\sqrt{3}+1$

Ans. **(1)**

Sol. $\frac{2+k^2z}{k+\bar{z}} = kz$

$|z|^2 k = 2$

$k = 2$

point p(2, 4); center (1, 2)

distance from circle

$(x-1)^2 + (y-2)^2 = 1$ is max.

if $(OP + r) = \sqrt{1+4}+1 = \sqrt{5}+1$

11. If $\vec{a}$ is nonzero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is:

(1) $\frac{1}{\sqrt{155}}\left(-7\hat{i}+9\hat{j}+5\hat{k}\right)$ (2) $\frac{1}{\sqrt{155}}\left(-7\hat{i}+9\hat{j}-5\hat{k}\right)$

(3) $\frac{1}{\sqrt{155}}\left(7\hat{i}+9\hat{j}+5\hat{k}\right)$ (4) $\frac{1}{\sqrt{155}}\left(7\hat{i}+9\hat{j}-5\hat{k}\right)$

Ans. **(3)**

Sol. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$a_1^2 + a_2^2 + a_3^2 = 1$

Let $\vec{b} = 2\vec{i} - \hat{j} + 2\hat{k}$, $\vec{c} = \vec{i} - 2\hat{j} - 2\hat{k}$

$\vec{d} = \hat{k}$

$\frac{\vec{a}.\vec{b}}{|b|} = \frac{\vec{a}.\vec{c}}{|c|} = \frac{\vec{a}.\vec{d}}{|d|}$

$\frac{2a_1 - a_2 + 2a_3}{3} = \frac{a_1 + 2a_2 - 2a_3}{3} = a_3$

By solving

$a_1 = \frac{7}{\sqrt{155}}$, $a_2 = \frac{9}{\sqrt{155}}$, $a_3 = \frac{5}{\sqrt{155}}$

12. Let A be the set of all functions $f : \mathbf{Z} \to \mathbf{Z}$ and R be a relation on A such that R = {(f, g) : $f(0) = g(1)$ and $f(1) = g(0)$}. Then R is:

(1) Symmetric and transitive but not reflective

(2) Symmetric but neither reflective nor transitive

(3) Reflexive but neither symmetric nor transitive

(4) Transitive but neither reflexive nor symmetric

Ans. **(2)**

Sol. R = {(f, g) : f(0) = g(1) and f(1) = g(0)}

Reflexive: (f, f) $\in$ R

= f(0) = f(1) and f(1) = f(0) $\rightarrow$ must hold

$\Rightarrow$ but this is not true for all function

so not reflexive

Symmetric: If (f, g) $\in$ R $\Rightarrow$ (g, f) $\in$ R

Now, g(0) = f(1) and g(1) = f(0) $\rightarrow$ true

$\therefore$ symmetric

Transitive : If(f, g) $\in$ R and (g, h) $\in$ R

$\Rightarrow$ (f, h) $\in$ R

Now (f, g) $\in$ R $\Rightarrow$ f(0) = g(1) and f(1) = g(0)

(g, h) $\in$ R $\Rightarrow$ g(0) = h(1) and g(1) = h(0)

For (f, h) $\in$ R we need f(0) = h(1) and f(1) = h(0)

Now f(0) = g(1) = h(0) and f(1) = g(0) = h(1)

Hence not transitive

13. For $\alpha, \beta, \gamma, \in \mathbf{R}$, if $\lim_{x\to 0}\frac{x^2\sin\alpha x+(\gamma-1)e^{x^2}}{\sin 2x-\beta x}=3$, then $\beta+\gamma-\alpha$ is equal to:

(1) 7 (2) 4

(3) 6 (4) –1

Ans. **(1)**

Sol. $$\lim_{x\to 10}\frac{x^2(\alpha x)+(\gamma-1)\left(1+\frac{x^2}{1}\right)}{2x-\frac{8x^3}{6}-\beta x}=3$$

$$\lim_{x\to 0}\frac{(\gamma-1)+(\gamma-1)x^2+\alpha x^3}{(2-\beta)x-\frac{4}{3}x^3}=3$$

$$\gamma-1,\ \beta=2,\ \frac{-3\alpha}{4}=+3\ \Rightarrow \alpha=-4$$

$\beta+\gamma-\alpha=7$

14. If the system of linear equations

$3x + y + \beta z = 3$

$2x + \alpha y - z = -3$

$x + 2y + z = 4$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is :

(1) 49 (2) 31

(3) 43 (4) 37

Ans. **(2)**

Sol. $$\Delta=\begin{vmatrix}3 & 1 & \beta\\ 2 & \alpha & -1\\ 1 & 2 & 1\end{vmatrix}=0$$

$3\alpha + 4\beta - \alpha\beta + 3 = 0$

$$\Delta_3=\begin{vmatrix}3 & 1 & 3\\ 2 & \alpha & -3\\ 1 & 2 & 4\end{vmatrix}=0$$

$9\alpha + 19 = 0$

$$\alpha=\frac{-19}{9},\ \beta=\frac{6}{11}$$

$\Rightarrow 22\beta - 9\alpha = 31$

15. Let $P_n = \alpha^n + \beta^n$, $n \in \mathbf{N}$. If $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

(1) $x^2 - x + 1 = 0$ (2) $x^2 + x - 1 = 0$

(3) $x^2 - x - 1 = 0$ (4) $x^2 + x + 1 = 0$

Ans. **(2)**

Sol. $\alpha^{10} + \beta^{10} = 123$

$\alpha + \beta = 1$

$\alpha^9 + \beta^9 = 76$

$\alpha^8 + \beta^8 = 47$

$P_{10} = P_9 + P_8$

$x^2 = x + 1 \Rightarrow x^2 - x - 1 = 0$

$\alpha + \beta = 1, \alpha\beta = -1$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{-1}=-1,\ \frac{1}{\alpha\beta}=-1$$

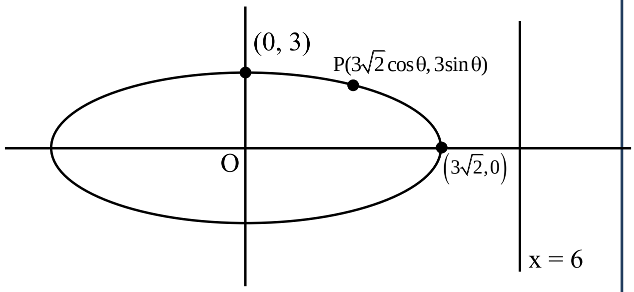
16. If S and S' are the foci of the ellipse $\frac{x^2}{18}+\frac{y^2}{9}=1$ and P be a point on the ellipse, then min(SP.S'P) + max(SP.S'P) is equal to :

(1) $3\left(1+\sqrt{2}\right)$ (2) $3\left(6+\sqrt{2}\right)$

(3) 9 (4) 27

Ans. **(4)**

Sol.



$PS + PS' = 2 \times 3\sqrt{2}$

$b^2 = a^2(1 - e^2) \Rightarrow 9 = 18(1 - e^2)$

$\Rightarrow e = \frac{1}{\sqrt{2}}$

Directrix $x = \frac{a}{e} = \frac{3\sqrt{2}}{\frac{1}{\sqrt{2}}} = 6$

$PS \cdot PS' = \left|\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta - 6)\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta + 6)\right|$

$= \frac{1}{2}\left|18\cos^2\theta - 36\right|$

$(PS \cdot PS')_{max} = 18$; $(PS \cdot PS')_{min} = 9$

sum = 27

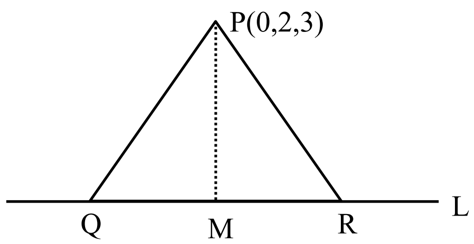
17. Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$, QR = 5 and the coordinates of the point P be (0, 2, 3). If the area of the triangle PQR is $\frac{m}{n}$ then :

(1) $m - 5\sqrt{21}n = 0$ (2) $2m - 5\sqrt{21}n = 0$

(3) $5m - 2\sqrt{21}n = 0$ (4) $5m - 21\sqrt{2}n = 0$

Ans. **(2)**

Sol.



QR = 5

$M(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

Drs of PM $\Rightarrow 5\lambda - 3, 2\lambda - 1, 3\lambda - 7$

Drs of line L $\Rightarrow 5, 2, 3$

$PM \perp L$

$\Rightarrow (5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 = 0$

$\Rightarrow \lambda = 1$

$\therefore M(2, 3, -1)$

$PM = \sqrt{4+1+16} = \sqrt{21}$

Area $= \frac{1}{2} \times 5 \times \sqrt{21} = \frac{m}{n}$

$2m - 5\sqrt{21}n = 0$

18. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5, 6 and 7 square units respectively. Then the area (in square units) of the ΔBCD is equal to :

(1) $\sqrt{340}$ (2) 12

(3) $\sqrt{110}$ (4) $7\sqrt{3}$

Ans. **(3)**

Sol. Ar(ΔBCD)

$= \sqrt{(Ar(\Delta ABC))^2 + (Ar(ACD))^2 + (Ar(\Delta ADB))^2}$

$= \sqrt{5^2 + 6^2 + 7^2}$

$= \sqrt{110}$

19. Let $a \in \mathbf{R}$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3.

If det ((a + 1)adj((a–1)A)) is 2^m3^n, m, n ∈ {0,1,2,…..20}, then m + n is equal to :

(1) 14 (2) 17

(3) 15 (4) 16

Ans. **(4)**

Sol. $A = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix} - I = \begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$

$|A| = -4 \Rightarrow 2 - 2a = -4 \Rightarrow a = 3$

$|(a + 1)\ \text{adj}\ (a - 1)A| = |4\ \text{adj}\ 3A|$

$= 4^3|\text{adj}\ 3A|$

$= 4^3 \times |3A|^{3-1} = 64|3A|^2$

$= 64 \times (3^3)^2\ |A|^2$

$= 2^6 \times 3^6 \times 16$

$2^m \times 3^n = 2^{10} \times 3^6$

$\therefore\ m = 10, n = 6$

$\Rightarrow\ m + n = 16$

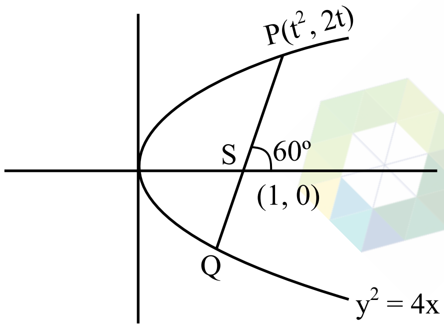
20. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x-axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

(1) 15 (2) 25

(3) 30 (4) 20

Ans. **(1)**

Sol.



$\tan 60° = \dfrac{2t - 0}{t^2 - 1} = \sqrt{3} \quad \Rightarrow t = \sqrt{3}$

$\therefore\ P(3, 2\sqrt{3})$

Circle :

$(x - 1)(x - 3) + (y - 0)(y - 2\sqrt{3}) = 0$

at $x = 0$

$\Rightarrow 3 + y^2 - 2\sqrt{3}\,y = 0$

$\Rightarrow y = \sqrt{3} = \alpha$

$5\alpha^2 = 15$

SECTION-B

21. Let $[\cdot]$ denote the greatest integer function. If $\displaystyle\int_0^{e^3} \left[\frac{1}{e^{x-1}}\right] dx = \alpha - \log_e 2$, then α^3 is equal to _____.

Ans. **(8)**

Sol. $f(x) = \dfrac{1}{e^{x-1}} = e^{1-x}$

$f(x) = 2$	$f(x) = 1$
$\dfrac{1}{e^{x-1}} = 2$	$x = 1$
$x = 1 - \ell n2$	

$f(0) = e^1 = 2.71$

$f(e^3) = e^{1-e^3} \in (0,1)$

$I = \int_0^{1-\ell n2} 2dx + \int_{1-\ell n2}^{1} 1dx + \int_1^{e^3} 0\,dx$

$= 2(1 - \ell n2 - 0) + 1(1 - 1 + \ell n2) + 0$

$\alpha - \ell n2 = 2 - \ell n2$

$\alpha = 2$

$\alpha^3 = 8$

22. Let $f : \mathbf{R} \to \mathbf{R}$ be a thrice differentiable odd function satisfying $f'(x) \geq 0,\ f''(x) = f(x), f(0) = 0, f'(0) = 3$. Then $9f(\log_e 3)$ is equal to _____.

Ans. **(36)**

Sol. $f''(x) = f(x)$

$\Rightarrow f'(x) \cdot f''(x) = f'(x) \cdot f(x)$

$\Rightarrow \dfrac{(f'(x))^2}{2} = \dfrac{(f(x))^2}{2} + C$

$\Rightarrow (f'(x))^2 = (f(x))^2 + C'$

$f(0) = 0,\ f'(0) = 3 \qquad \Rightarrow C' = 9$

$\therefore\ (f'(x))^2 = (f(x))^2 + 9$

$f'(x) = \sqrt{(f(x))^2 + 9} \qquad \because\ f'(x) \geq 0$

$\displaystyle\int \frac{dy}{\sqrt{y^2 + 9}} = \int dx \Rightarrow \ell n\left|y + \sqrt{y^2 + 9}\right| = x + C$

$\Rightarrow f(0) = 0 \ \Rightarrow C = \ell n3$

$\Rightarrow\ y + \sqrt{y^2 + 9} = 3e^x$

at $x = \ell n3$; $y = 4$

$\therefore\ 9f(\ell n3) = 36$

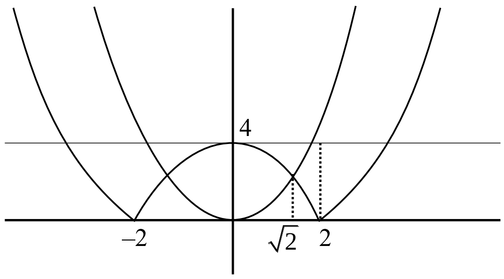
23. If the area of the region

$\left\{(x,y): \left|4-x^2\right| \le y \le x^2, y \le 4, x \ge 0\right\}$

is $\left(\frac{80\sqrt{2}}{\alpha} - \beta\right)$, $\alpha, \beta \in \mathbf{N}$, then $\alpha + \beta$ is equal to _____.

Ans. **(22)**

Sol.



$$A = \int_0^4 \sqrt{4+y}\,dy - \int_0^2 \sqrt{4-y}\,dy - \int_2^4 \sqrt{y}\,dy$$

$$= \left(\frac{(4+y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4 + \left(\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 - \left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$$

$$\frac{80\sqrt{2}}{3} - 16 = \frac{40\sqrt{2}}{3} - 16$$

$\alpha = 6, \beta = 16$

$\alpha + \beta = 22$

24. Three distinct numbers are selected randomly from the set $\{1,2,3,\ldots\ldots,40\}$. If the probability, that the selected numbers are in an increasing G.P. is $\frac{m}{n}$, $\gcd(m,n) = 1$, then $m + n$ is equal to ____.

NTA Ans. (4949)

Allen Ans. (2477)

Sol. $1 \le a < ar < ar^2 \le 40$

(If $r \in N$)

<u>If r = 2</u>

$1 \le a < 2a < 4a \le 40$

$a \in \{1, \ldots\ldots, 10\}$ (10 GP)

<u>If r = 3</u>

$1 \le a < 3a < 9a \le 40$

$a \in \{1, 2, 3, 4\}$ (4 GP)

<u>If r = 4</u>

$1 \le a < 4a < 16a \le 40$

$a \in \{1, 2\}$ (2 GP)

<u>If r = 5</u>

$1 \le a < 5a < 25a \le 40$

$a \in \{1\}$ (1 GP)

<u>If r = 6</u>

$1 \le a < 6a < 36a \le 40$

$a \in \{1\}$ (1 GP)

$\left(P = \frac{18}{9880} = \frac{9}{4940}\right)$ as per NTA for $r \in N$

$m + n = 4949$

If <u>$r \notin N$</u> (also possible)

$r = \frac{3}{2}$

$ar^2 = \frac{9a}{4}$; $a = 4k$

$\left.\begin{array}{l}(4,6,9)\\(8,12,18)\\(12,18,27)\\(16,24,36)\end{array}\right\}$ 4GP

$r = \frac{5}{2}$ $ar^2 = \frac{25a}{4}$; $a = 4k$

(4, 10, 25) ....(1) GP

$r = \frac{4}{3}$ $ar^2 = \frac{16a}{9} \rightarrow a = 9k$

(9, 12, 16), (18, 24, 32) ....(2) GP

$r = \frac{5}{3}$ $ar^2 = \frac{25a}{9}$; $a = 9k$

(9, 15, 25) ....(1) GP

$r = \frac{5}{4}$ $ar^2 = \frac{25a}{16}$; $a = 16k$

(16, 20, 25) ....(1) GP

$r = \frac{6}{5}$ $ar^2 = \frac{36a}{25}$; $a = 25k$

(25, 30, 36) ....(1) GP

Total = 18 + 10 = 28

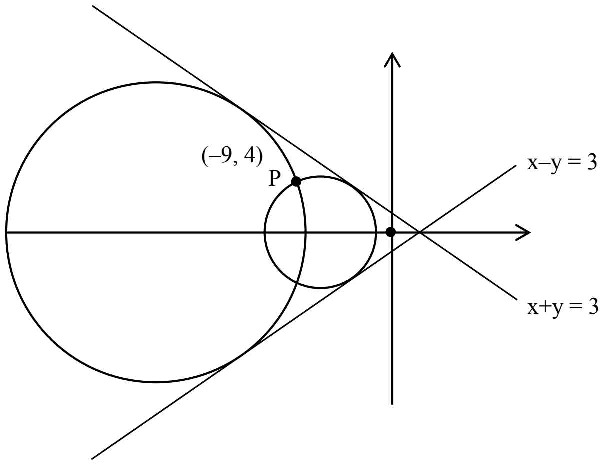
$$P = \frac{28}{{}^{40}C_3} = \frac{28}{9880} = \frac{7}{2470}$$

$m + n = 2477$

25. The absolute difference between the squares of the radii of the two circles passing through the point (–9, 4) and touching the lines $x + y = 3$ and $x - y = 3$, is equal to _____.

Ans. (768)

Sol.



Centre (a, 0)

$$r = \left|\frac{a - 0 - 3}{\sqrt{2}}\right|$$

circle $(x - a)^2 + y^2 = \left(\frac{a-3}{\sqrt{2}}\right)^2$

passes through (–9, 4)

$2(a^2 + 18a + 81 + 16) = (a^2 - 6a + 9)$

$a^2 + 42a + 185 = 0$

$(a + 37)(a + 5) = 0$

$\Rightarrow a = -37, -5$

$$r_1 = \left|\frac{-37-3}{\sqrt{2}}\right| = 20\sqrt{2}$$

$$r_2 = \left|\frac{-5-3}{\sqrt{2}}\right| = 4\sqrt{2}$$

$$\left|r_1^2 - r_2^2\right| = |800 - 32| = 768$$