JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON THURSDAY 03rd APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f : R \to R$ be a function defined by $f(x) = \left||x+2|-2|x|\right|$. If m is the number of points of local minima and n is the number of points of local maxima of f, then m + n is

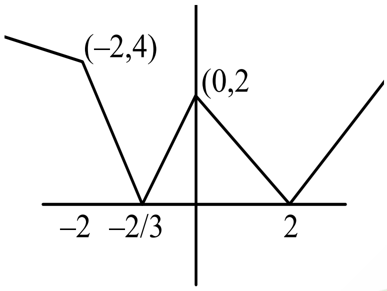
(1) 5 (2) 3

(3) 2 (4) 4

Ans. **(2)**

Sol. $f(x) = \left||x+2|-2|x|\right|$

Critical points, $0, -2, 2, -\frac{2}{3}$



No. of maxima = 1

No. of minima = 2

option (2)

2. Each of the angles β and γ that a given line makes with the positive y– and z–axes, respectively, is half of the angle that this line makes with the positive x-axes. Then the sum of all possible values of the angle β is

(1) $\frac{3\pi}{4}$ (2) π

(3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{2}$

Ans. **(1)**

Sol. $\beta = \frac{\alpha}{2}, \gamma = \frac{\alpha}{2}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha + 2\cos^2\frac{\alpha}{2} = 1$

$\cos^2\alpha + \cos\alpha = 0$

$\cos\alpha(\cos\alpha + 1) = 0$

$\cos\alpha = 0, -1$

$\alpha = \frac{\pi}{2}, \pi$

Now $\beta = \frac{\alpha}{2} \Rightarrow \frac{\pi}{4}, \frac{\pi}{2}$

so sum is $\frac{3\pi}{4}$

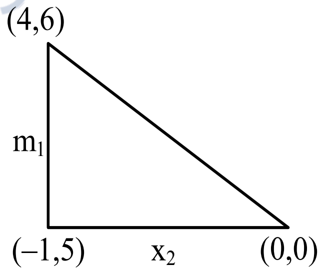
3. If the four distinct points (4, 6), (–1,5), (0,0) and (k, 3k) lie on a circle of radius r, then $10k + r^2$ is equal to

(1) 32 (2) 33

(3) 34 (4) 35

Ans. **(4)**

Sol.



$m_1m_2 = -1$ so right angle equation circle is

$(x-4)(x-0) + (y-6)(y-0) = 0$

$x^2 + y^2 - 4x - 6y = 0$

(k,3k) lies on it so

$k^2 + 9k^2 - 4k - 18k = 0$

$10k^2 - 22k = 0$

$k = 0, \frac{11}{5}$

k = 0 is not possible so $k = \frac{11}{5}$

also $r = \sqrt{4+9} = \sqrt{13}$

so $10k + r^2 = 10.\frac{11}{5} + \left(\sqrt{13}\right)^2 = 35$

4. Let the Mean and Variance of five observations $x_1 = 1$, $x_2 = 3$, $x_3 = a$, $x_4 = 7$ and $x_5 = b$, $a > b$, be 5 and 10 respectively. Then the Variance of the observations $n + x_n$, n = 1, 2, …….. 5 is

(1) 17 (2) 16.4

(3) 17.4 (4) 16

Ans. **(4)**

Sol. $\bar{x} = \frac{\sum x_i}{n} = \frac{1+3+a+7+b}{5} = 5$

$a + b = 14$

$\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \frac{1^2 + 3^2 + a^2 + 7^2 + b^2}{5} - 25 = 10$

$a^2 + b^2 = 116$

$a > b \quad a = 10 \quad b = 4$

$n + x_n : 2,5,13,11,9$

$\sigma^2 = \frac{2^2 + 5^2 + 13^2 + 11^2 + 9^2}{5} - \left(\frac{2+5+13+11+9}{5}\right)^2$

$= 80 - 64 = 16$

option 4

5. Consider the lines $x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$, λ being a parameter, all passing through a point P. One of these lines (say L) is farthest from the origin. If the distance of L from the point (3, 6) is d, then the value of d^2 is

(1) 20 (2) 30

(3) 10 (4) 15

Ans. **(1)**

Sol. $x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$

$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

intersection of family of lines

P(1,2)

Let Q(3,6)

$d = PQ = \sqrt{2^2 + 4^2} = \sqrt{20}$

$d^2 = 20$

option (1)

6. Let A = {–2,–1,0, 1, 2, 3}. let R be a relation on A defined by xRy if and only if y = max{x, 1}. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then l + m + n is equal to

(1) 12 (2) 11

(3) 13 (4) 14

Ans. **(1)**

Sol. A = {–2,–1,0,1,2,3}

R = {(–2,1),(–1,1), (0,1), (1,1), (2,2), (3,3)}

$\ell = 6$

m = 3

n = 3

$\ell + m + n = 12$

7. let the equation x(x + 2)(12–k) = 2 have equal roots. Then the distance of the point $\left(k, \frac{k}{2}\right)$ from the line 3x + 4y + 5 = 0 is

(1) 15 (2) $5\sqrt{3}$

(3) $15\sqrt{5}$ (4) 12

Ans. **(1)**

Sol. $(x^2 + 2x)(12 - k) = 2$

$\lambda x^2 + 2\lambda x - 2 = 0 \quad k \neq 12 \quad \text{Let } 12 - k = \lambda$

D = 0

$4\lambda^2 + 8\lambda = 0$

$\lambda = 0$ or $\lambda = -2$

$\Rightarrow 12 - k = -2$

k = 14

So $P\left(k, \frac{k}{2}\right) = (14, 7)$

$d = \left|\frac{3 \times 14 + 4 \times 7 + 5}{5}\right| = 15$

option (1)

8. Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O. In the first quadrant, $P_1, P_2, \ldots P_{12}$ are 12 points on line L_1 and $Q_1, Q_2, \ldots Q_9$ are 9 points on line L_2. Then the total number of triangles, that can be formed having vertices at three of the 22 points O, $P_1, P_2, \ldots P_{12}$, $Q_1, Q_2, \ldots Q_9$, is:

(1) 1080 (2) 1134

(3) 1026 (4) 1188

Ans. (2)

Sol. Total number of Δare

$= {}^9C_1{}^{12}C_2 + {}^9C_2{}^{12}C_1 + {}^1C_1{}^9C_1{}^{12}C_1$

$= 594 + 432 + 108$

$= 1134$

9. The integral $\int_0^{\pi} \frac{8x\,dx}{4\cos^2 x + \sin^2 x}$ is equal to

(1) $2\pi^2$ (2) $4\pi^2$

(3) π^2 (4) $\frac{3\pi^2}{2}$

Ans. (1)

Sol. $I = \int_0^{\pi} \frac{8x\,dx}{4\cos^2 x + \sin^2 x}$

$I = \int_0^{\pi} \frac{8(\pi - x)dx}{4\cos^2 x + \sin^2 x}$

$2I = 8\pi\int_0^{\pi} \frac{dx}{4\cos^2 x + \sin^2 x}$

$2I = 8\pi \times 2\int_0^{\pi/2} \frac{\sec^2 x}{4 + \tan^2 x}dx$

$I = 8\pi\int_0^{\infty} \frac{dt}{4 + t^2} = 8\pi \times \frac{1}{2}\left(\tan^{-1}\frac{t}{2}\right]_0^{\infty}$

$= 4\pi \times \frac{\pi}{2} = 2\pi^2$

option (1)

10. Let f be a function such that $f(x) + 3f\left(\frac{24}{x}\right) = 4x,\ x \neq 0$. Then f(3) + f(8) is equal to

(1) 11 (2) 10

(3) 12 (4) 13

Ans. (1)

Sol. $f(x) + 3\,f\left(\frac{24}{x}\right) = 4x$

Put x = 3   f(3) + 3f(8) = 12

Put x = 8   f(8) + 3f(3) = 32

Add both   4(f(3) + f(8)) = 44

f(3) + f(8) = 11

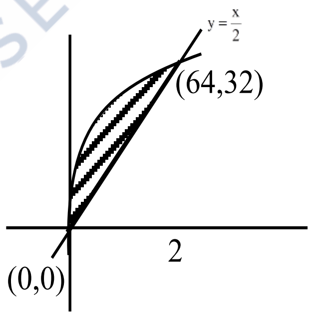
11. The area of the region $\{(x,y) : |x - y| \leq y \leq 4\sqrt{x}\}$ is

(1) 512 (2) $\frac{1024}{3}$

(3) $\frac{512}{3}$ (4) $\frac{2048}{3}$

Ans. (2)

Sol.



$|x - y| \leq y \leq 4\sqrt{x}$

Now $y = |x - y|$

$y^2 = (x - y)^2$

$\Rightarrow y = \frac{x}{2}$ and x = 0

Now area $= \int_0^{64}\left(4\sqrt{x} - \frac{x}{2}\right)dx$

$= \left[\frac{4x^{3/2}}{3/2} - \frac{x^2}{4}\right]_0^{64} = \frac{8}{3}.8^3 - \frac{64^2}{4} = 64^2\left(\frac{1}{12}\right)$

$= \frac{1024}{3}$

12. If the domain of the function $f(x) = \log_7(1 - \log_4(x^2 - 9x + 18))$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $\alpha + \beta + \gamma + \delta$ is equal to

(1) 18 (2) 16

(3) 15 (4) 17

Ans. (1)

Sol. Domain $1 - \log_4(x^2 - 9x + 18) > 0$

Also $x^2 - 9x + 18 > 0$

$(x - 3)(x - 6) > 0$

$x \in (-\infty, 3) \cup (6, \infty)$ ....(1)

also $x^2 - 9x + 18 < 4$

$x^2 - 9x + 14 < 0$

$x \in (2,7)$ ....(2)

$(1) \cap (2)$ $(2,3) \cup (6,7) = (\alpha,\beta) \cup (\gamma,\delta)$

$\Rightarrow \alpha + \beta + \gamma + \delta = 18$

13. If the probability that the random variable X takes the value x is given by $P(X = x) = k(x + 1)3^{-x}$, $x = 0,1,2,3......$, where k is a constant, then $P(X \geq 3)$ is equal to

(1) $\frac{7}{27}$ (2) $\frac{4}{9}$

(3) $\frac{8}{27}$ (4) $\frac{1}{9}$

Ans. (4)

Sol. $\sum_{x=0}^{\infty} k(x+1)3^{-x} = 1$

$\Rightarrow \frac{1}{k} = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots$ (i)

$\frac{1}{3k} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \ldots.$ ...(ii)

(i)- (ii) $\Rightarrow \frac{1}{k} - \frac{1}{3k} = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots$

$\Rightarrow k = \frac{4}{9}$

$P(x \geq 3) = 1 - P(x = 0) - P(x = 1) - P(x = 2)$

$= 1 - k\left(1 + \frac{2}{3} + \frac{3}{9}\right) = \frac{1}{9}$

14. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 3(\tan^2 x)\, y + 3y = \sec^2 x$, $y(0) = \frac{1}{3} + e^3$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$

(3) $\frac{4}{3} + e^3$ (4) $\frac{2}{3} + e^3$

Ans. (2)

Sol. $\frac{dy}{dx} + 3(\sec^2 x)y = \sec^2 x, y(0) = \frac{1}{3} + e^3$

$\text{If} = e^{3\int \sec^2 x dx} = e^{3\tan x}$

$\therefore$ Solution is

$e^{3\tan x} y = \int e^{3\tan x} \sec^2 x dx$

$e^{3\tan x} y = \frac{e^{3\tan x}}{3} + c$

$\because y(0) = \frac{1}{3} + e^3 \Rightarrow c = e^3$

$\therefore y\left(\frac{\pi}{4}\right) = \frac{\frac{e^3}{3} + e^3}{e^3} = \frac{4}{3}$

15. If $z_1, z_2, z_3 \in C$ are the vertices of an equilateral triangle, whose centroid is z_0, then $\sum_{k=1}^{3}(z_k - z_0)^2$ is equal to

(1) 0 (2)

(3) i (4) –i

Ans. (1)

Sol. $z_1 + z_2 + z_3 = 3z_0$

$(z_1 + z_2 + z_3)^2 = 9z_0^2$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 + 2\left(z_1^2 + z_2^2 + z_3^2\right) = 9z_0^2$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = 3z_6^2$

$\sum_{k=1}^{3}(z_k - z_0)^2 = (z_1 - z_0)^2 + (z_2 - z_0)^2 + (z_3 - z_0)^2$

$= z_1^2 + z_2^2 + z_3^2 + 3z_0^2 - 2(z_1 + z_2 + z_3)z_0$

$= 6z_0^2 - 6z_0^2$

$= 0$

16. The number of solutions of equation $\left(4-\sqrt{3}\right)\sin x - 2\sqrt{3}\cos^2 x = -\frac{4}{1+\sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$ is

(1) 4 (2) 3
(3) 6 (4) 5

Ans. (4)

Sol. $\left(4-\sqrt{3}\right)\sin x - 2\sqrt{3}\cos^2 x = \frac{-4}{1+\sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$

$\Rightarrow \left(4-\sqrt{3}\right)\sin x - 2\sqrt{3}\left(1-\sin^2 x\right) = 2\left(1-\sqrt{3}\right)$

$\Rightarrow 2\sqrt{3}\sin^2 x + 4\sin x - \sqrt{3}\sin x - 2 = 0$

$\Rightarrow \left(2\sin x - 1\right)\left(\sqrt{3}\sin x + 2\right) = 0$

$\Rightarrow \sin x = \frac{1}{2}$

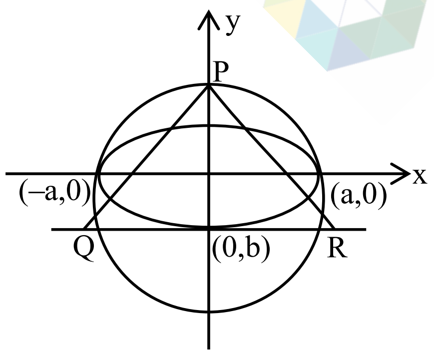
$\therefore$ Number of solution = 5

17. Let C be the circle of minimum area enclosing the ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ with eccentricity $\frac{1}{2}$ and foci$(\pm 2, 0)$. Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length 29 is parallel to the major axis of E and contains the point of intersection of E with the negative y-axis. Then the maximum area of the triangle PQR is :

(1) $6\left(3+\sqrt{2}\right)$ (2) $8\left(3+\sqrt{2}\right)$
(3) $6\ 2+\sqrt{3}$ (4) $8\ 2+\sqrt{3}$

Ans. (4)

Sol.



Area of ΔPQR

$= \frac{1}{2}(2a)(a\sin\theta + b)$

$\therefore$ maximum area = $a(a + b)$

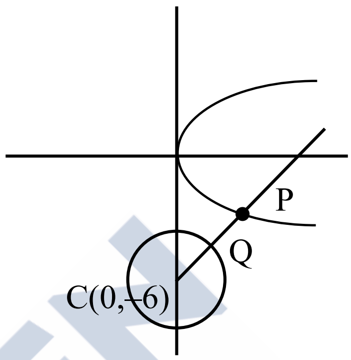
$= 4(4 + 2\sqrt{3}) = 8\left(2+\sqrt{3}\right)$

18. The shortest distance between the curves $y^2 = 8x$ and $x^2 + y^2 + 12y + 35 = 0$ is :

(1) $2\sqrt{3} - 1$ (2) $\sqrt{2}$
(3) $3\sqrt{2} - 1$ (4) $2\sqrt{2} - 1$

Ans. (4)

Sol.



Equation of normal to parabola

$y^2 = 8x$ is $y = mx - 4m - 2m^3$

passes through (0,–6) we get

$-6 = -4m - 2m^3$

$\Rightarrow m^3 + 2m - 3 = 0$

$\Rightarrow (m-1)(m^2 + m + 3) = 0 \Rightarrow m = -1$

$P = (am^2, -2am) = (2, -4)$

$\therefore$ Shortest distance = PC – r

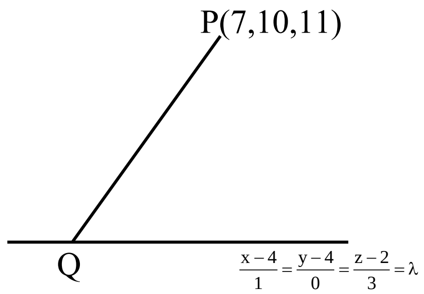
$= \left(2\sqrt{2} - 1\right)$

19. The distance of the point (7, 10, 11) from the line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

(1) 18 (2) 14
(3) 12 (4) 16

Ans. (2)

Sol.



$(\lambda+4, 4, 3\lambda+2)$

$\because$ line PQ is parallel to line $\frac{x-9}{2}=\frac{y-3}{3}=\frac{z-17}{6}$

$\therefore\ \frac{\lambda-3}{2}=\frac{-6}{3}=\frac{3\lambda-9}{6} \Rightarrow \lambda=-1$

$Q=(3,4,-1)$

$\therefore\ PQ=\sqrt{16+36+144}=14$

20. The sum $1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+\frac{1+3+5+7}{4!}+\ldots$ upto ∞ terms, is equal to

(1) $6e$ (2) $4e$

(3) $3e$ (4) $2e$

Ans. **(4)**

Sol. $S=1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+\ldots$

$=\sum_{r=1}^{\infty}\frac{r^2}{r!}$

$=\sum_{r=1}^{\infty}\frac{(r-1+1)}{(r-1)!}=\sum_{r=2}^{\infty}\frac{1}{(r-2)!}+\sum_{r=1}^{\infty}\frac{1}{(r-1)!}$

$=2e$

SECTION-B

21. Let I be the identity matrix of order 3 × 3 and for the matrix $A=\begin{bmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{bmatrix}$, $|A|=-1$. Let B be the inverse of the matrix $adj(A\ adj(A^2))$. Then $|(\lambda B+1)|$ is equal to _______

Ans. **(38)**

Sol. $|A|=\begin{vmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{vmatrix}=-1$

$\lambda(16)-2(-34)+3(-39)=-1$

$16\lambda=48 \Rightarrow \lambda=3$

$B^{-1}=adj(A.adj\,(A^2))$

Let $C=A.\ adj\,(A^2)$

$AC=A^2\ adj(A^2)=|A|^2.I=I \Rightarrow C=A^{-1}$

Now $B^{-1}=adj(A^{-1})=B=adj(A)$

Now $\lambda B+I \Rightarrow 3B+I$

Let $P=3B+I$

$P=3adj\,(A)+I$

$AP=3Aadj(A)+A$

$AP=3|A|.I+A$

$AP=A-3I$

$|AP|=|A-3I|$

$|A|.\ |P|=\begin{vmatrix}0 & 2 & 3\\ 4 & 2 & 6\\ 7 & -1 & -1\end{vmatrix}=38$

$|P|=-38$

22. Let $(1+x+x^2)^{10}=a_0+a_1x+a_2x^2+\ldots+a_{20}x^{20}$. If $(a_1+a_3+a_5+\ldots+a_{19})-11a_2=121k$, then k is equal to_____.

Ans. **(239)**

Sol. $(1+x+x^2)^{10}=a_0+a_1x+a_2x^2+\ldots+a_{20}x^{20}$

$\therefore\ 3^{10}=a_0+a_1+a_2+\ldots+a_{20}$ ....(i)

$1=a_0-a_1+a_2\ \ldots\ldots+a_{20}$ ...(ii)

(i) - (ii) $\Rightarrow a_1+a_3+\ldots+a_{19}=\frac{3^{10}-1}{2}=29524$

Also $\{1+x(1+x)\}^{10}=1$

$+\ {}^{10}C_1x(1+x)+{}^{10}C_2x^2(1+x)^2+\ldots$

$\therefore\ a_2={}^{10}C_1+{}^{10}C_2=55$

$\therefore\ \frac{(a_1+a_3+\ldots+a_{19})-11a_2}{121}=239$

23. If $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}} = p$, then $96 \log_e p$ is equal to ___

Ans. **(32)**

Sol. $P = \lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

$\Rightarrow P = e^{\lim_{x\to 0}\left(\frac{\tan x - x}{x^3}\right)}$

$= e^{\lim_{x\to 0}\frac{\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+.....-x\right)}{x^3}}$

$= e^{1/3}$

$\therefore 96\log_e^{P} = 96 \times \frac{1}{3} = 32$

24. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b}\times\vec{d} = \vec{c}\times\vec{d}$ and $\vec{a}\cdot\vec{d} = 4$. Then $|(\vec{a}\times\vec{d})|^2$ is equal to ______.

Ans. **(128)**

Sol. $\vec{b}\times\vec{d} = \vec{c}\times\vec{d}$ -and $\vec{a}\,.\,\vec{d} = 4$

$\Rightarrow \vec{d} = \lambda\left(\vec{b}-\vec{c}\right) = \lambda\left(\hat{i}-2\hat{j}+\hat{k}\right)$

$\because\ \vec{a}.\vec{d} = 4 \Rightarrow \lambda = -2$

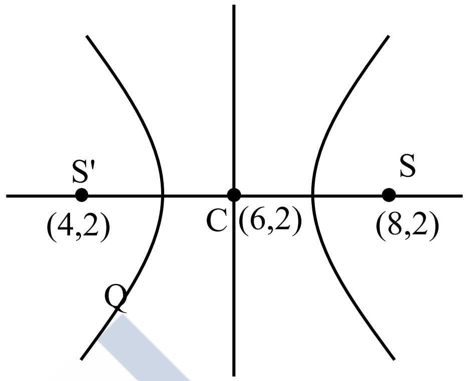
Also. $\left|\vec{a}\times\vec{d}\right|^2 + \left|\vec{a}\,.\,\vec{d}\right|^2 = \left|\vec{a}\right|^2\left|\vec{d}\right|^2$

$\Rightarrow \left|\vec{a}\times\vec{d}\right|^2 = 6\times4\times6-16 = 128$

25. If the equation of the hyperbola with foci (4, 2) and (8, 2) is $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, then $\alpha + \beta + \gamma$ is equal to ______.

Ans. **(141)**

Sol.



Equation of hyperbola is

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{4-a^2} = 1$$

$$\Rightarrow \left(4-a^2\right)(x-6)^2 - a^2(y-2)^2 = a^2\left(4-a^2\right)$$

comparing with $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, we get

$a^2 = 1$ and $\alpha = 36$, $\beta = 4$ and $\gamma = 101$

$\therefore\ \alpha + \beta + \gamma = 141$