JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON THURSDAY 03rd APRIL 2025) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let A be a matrix of order 3 x 3 and $|A| = 5$. If $\left|2\text{adj}(3A\,\text{adj}(2A))\right| = 2^{\alpha}.3^{\beta}.5^{\gamma}$ $\alpha, \beta, \gamma \in N$ then $\alpha + \beta + \gamma$ is equal to

(1) 25 (2) 26

(3) 27 (4) 28

Ans. **(3)**

Sol. |2 adj (3A adj(2A))|

$2^3.|3A\ \text{adj}(2A)|^2$

$2^3.(3^3)^2.|A|^2.|\text{adj}(2A)|^2$

$2^3.3^6.|A|^2.(|2A|^2)^2$

$2^3.3^6.|A|^2[(2^3)^2.|A|^2]^2$

$2^3.3^6.|A|^2.2^{12}.|A|^4$

$2^{15}.3^6.|A|^6$

$2^{15}.3^6.5^6 = 2^{\alpha}.3^{\beta}.5^{\gamma}$

$\alpha = 15, \quad \beta = 6, \quad \gamma = 6$

$\alpha + \beta + \gamma = 27$

Option (3)

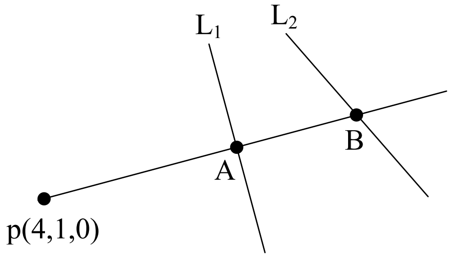
2. Let a line passing through the point (4,1,0) intersect the line $L_1; \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point A (α, β, γ) and the line $L_2 : x - 6 = y = -z + 4$ at the point B (a, b, c). Then $\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$ is equal to

(1) 8 (2) 16

(3) 12 (4) 6

Ans. **(1)**

Sol.



$L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = p$

$L_2 = \frac{x-6}{1} = \frac{y}{1} = \frac{z-4}{-1} = q$

A(2p + 1, 3p + 2, 4p + 3)

B(q + 6, q, 4 – q)

D.R. of PA = 2p – 3, 3P + 1, 4p + 3

D.R. of PB = q + 2, q – 1, 4 – q

$\frac{2p-3}{q+2} = \frac{3p+1}{q-1} = \frac{4p+3}{4-q}$

2pq – 2p – 3q + 3 = 3pq + 6p + q + 2

pq + rp + 4q – 1 = 0 ....(1)

12p – 3pq + 4 – q = 4pq + 3q – 4p – 3

7pq – 16p + 4q – 7 = 0 ....(2)

8p – 2pq – 12 + 3q = 4pq + 8p + 3q + 6

6pq = –18 $\therefore$ $\boxed{pq = -3}$

8p + 4q = 4 $\Rightarrow$ 2p + q = 1

–21 – 16p + 4q – 7 $\Rightarrow$ 4p – q = –7

16p – 4q = –28 $\therefore$ p = –1, q = 3

A(–1, –1, –1) B(9, 3, 1)

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 1(-1+9) = 8$$

Option (1)

3. Let α and β be the roots of $x^2+\sqrt{3}x-16=0$, and γ and δ be the roots of $x^2+3x-1=0$. If $P_n=\alpha^n+\beta^n$ and $Q_n=\gamma^n+\delta^n$, then

$\dfrac{P_{25}+\sqrt{3P_{24}}}{2P_{23}}+\dfrac{Q_{25}-Q_{23}}{Q_{24}}$ is equal to

(1) 3 (2) 4

(3) 5 (4) 7

Ans. (3)

Sol. $x^2+\sqrt{3}x-16=0 < \begin{matrix}\alpha\\\beta\end{matrix}$ $\quad P_n=\alpha^n+\beta^n$

$$P_n+\sqrt{3}P_{n-1}-16P_{n-2}=0$$

$$P_{25}+\sqrt{3}P_{24}-16P_{23}=0$$

$$\therefore \frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}=8$$

Similarly

$x^2+3x-1=0 < \begin{matrix}\gamma\\\delta\end{matrix}$ $\quad Q_n=\gamma^n+\delta^n$

$$Q_{25}-Q_{23}=\gamma^{25}+\delta^{25}-\gamma^{23}-\delta^{23}$$
$$=\gamma^{23}(\gamma^2-1)+\delta^{23}(\delta^2-1)$$
$$=\gamma^{23}(-3\gamma)+\delta^{23}(-3\gamma)$$
$$=-3[\gamma^{24}+\delta^{24}]$$
$$=-3\,Q_{24}$$

$$\therefore \frac{Q_{25}-Q_{23}}{Q_{24}}=-3$$

$$\frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}+\frac{Q_{25}-Q_{23}}{Q_{24}}=8-3=5$$

Option (3)

4. The sum of all rational terms in the expansion of $\left(2+\sqrt{3}\right)^8$ is

(1) 16923 (2) 3763

(3) 33845 (4) 18817

Ans. (4)

Sol. $S=\left(2+\sqrt{3}\right)^8$

For sum of rational terms

$$={}^8C_0(2)^8+{}^8C_2(2)^6.\left(\sqrt{3}\right)^2+{}^8C_4(2)^4\left(\sqrt{3}\right)^4+{}^8C_6(2)^2\left(\sqrt{3}\right)^6+{}^8C_8\left(\sqrt{3}\right)^8$$

$=2^8+28\times2^6.3+70.2^4.9+28.2^2.27+81$

$=256+5376+10080+3024+81$

$=18817$

Option (4)

5. Let $A=\{-3,-2,-1,0,1,2,3,\}$. Let R be a relation on A defined by xRy if and only if $0\le x^2+2y\le4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. then $l+m$ is equal to

(1) 19 (2) 20

(3) 17 (4) 18

Ans. (4)

Sol. $A=\{-3,-2,-1,0,1,2,3\}$

$-2y\le x^2\le 4-2y$

$y=-3$	$6\le x^2\le10$	$\Rightarrow x\in\{-3,3\}$
$y=-2$	$4\le x^2\le8$	$\Rightarrow x\in\{-2,2\}$
$y=-1$	$2\le x^2\le6$	$\Rightarrow x\in\{-2,2\}$
$y=0$	$0\le x^2\le4$	$\Rightarrow x\in\{-2,-1,0,1,2\}$
$y=1$	$-2\le x^2\le2$	$\Rightarrow x\in\{-1,0,1\}$
$y=2$	$-4\le x^2\le0$	$\Rightarrow x\in\{0\}$
$y=3$	$-6\le x^2\le-2$	$\Rightarrow$ No x-Exist

R = {(–3,–3) (–3,3), (–2,–2) (–2,2) (–1,–2) (–1,2) (0,–2) (0,–1) (0, 0) (0,1) (0,2) (1, –1) (1,0) (1,1) (2,0)}

$\therefore \ell=15$

To make it reflexive we will add

{(–1, –1), (2,2), (3,3)} $\quad\therefore m=3$

$\therefore \ell+m=15+3=18$

Option (4)

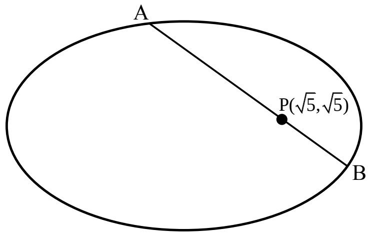
6. A line passing through the point $P\left(\sqrt{5},\sqrt{5}\right)$ intersects the ellipse $\frac{x^2}{36}+\frac{y^2}{25}=1$ at A and B such that $(PA).(PB)$ is maximum. Then $5(PA^2+PB^2)$ is equal to :

(1) 218 (2) 377

(3) 290 (4) 338

Ans. **(4)**

Sol. Given ellipse is $\frac{x^2}{36}+\frac{y^2}{25}=1$



Any point on line AB can be assumed as

$Q(\sqrt{5}+r\cos\theta,\sqrt{5}+r\sin\theta)$

Putting this in equation of ellipse, we get

$25(\sqrt{5}+r\cos\theta)^2+36(\sqrt{5}+r\sin\theta)^2=900$

Simplifying, we get

$r^2(25\cos^2\theta+36\sin^2\theta)+2\sqrt{5}r(25\cos\theta+36\sin\theta)-595=0$

$|r| = PA, PB$

Thus, $PA\cdot PB=\frac{595}{25\cos^2\theta+36\sin^2\theta}=\frac{595}{25+11\sin^2\theta}$

= maximum, if $\sin^2\theta = 0$

This means line AB must be parallel to x-axis

$\Rightarrow y_A = y_B = \sqrt{5}$

Putting $y=\sqrt{5}$ in equation of ellipse, we get

$\frac{x^2}{36}+\frac{1}{5}=1\Rightarrow x^2=36.\frac{4}{5}$

Hence,

$PA^2+PB^2=\left(\sqrt{5}-\frac{12}{\sqrt{5}}\right)^2+\left(\sqrt{5}+\frac{12}{\sqrt{5}}\right)^2$

$=2\left(5+\frac{144}{5}\right)=\frac{338}{5}$

$5(PA^2 + PB^2) = 338$

7. The sum 1 + 3 + 11 + 25 + 45 + 71 + .. upto 20 terms, is equal to

(1) 7240 (2) 7130

(3) 6982 (4) 8124

Ans. **(1)**

Sol. Given sum is

$S_n = 1 + 3 + 11 + 25 + 45 + 71 + ... + T_n$

First order differences are in A.P.

Thus, we can assume that

$T_n = an^2 + bn + c$

Solving $\left\{\begin{array}{c} T_1=1=a+b+c \\ T_2=3=4a+2b+c \\ T_3=11=9a+3b+c \end{array}\right\}$,

we get a = 3, b = –7, c = 5

Hence, general term of given series is

$T_n = 3n^2 - 7n + 5$

Hence, required sum equals

$$\sum_{n=1}^{n=20}(3n^2-7n+5)=3\left(\frac{20\cdot21\cdot41}{6}\right)-7\left(\frac{20\cdot21}{2}\right)+5(20)=7240$$

8. If the domain of the function $f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is $[\alpha, \beta)$, then $\alpha^2 + 4\beta$ is equal to

(1) 5 (2) 4

(3) 3 (4) 7

Ans. **(2)**

Sol. Given function is

$f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$

For domain, the conditions are

$\frac{2x-3}{5+4x}>0$ and $\left|\frac{4+3x}{2-x}\right|\le1$

Now, $\frac{2x-3}{5+4x}>0 \Rightarrow x\in\left(-\infty,-\frac{5}{4}\right)\cup\left[\frac{3}{2},\infty\right)$

and $-1\le\frac{4+3x}{2-x}\le1$

$\Rightarrow\left(-1\le\frac{4+3x}{2-x}\right)\cap\left(\frac{4+3x}{2-x}\le1\right)$

$\Rightarrow\left(\frac{6+2x}{2-x}\ge0\right)\cap\left(\frac{2+4x}{2-x}\le0\right)$

$\Rightarrow \frac{6+2x}{2-x} \cdot \frac{2+4x}{2-x} \leq 0$

$\Rightarrow x \in \left[-3, -\frac{1}{2}\right]$

Hence, we get the domain of f as $x \in \left[-3, -\frac{5}{4}\right)$

This means that $\alpha = -3, \beta = -\frac{5}{4}$

Thus, $\alpha^2 + 4\beta = 9 - 5 = 4$

9. If $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta$, $\alpha, \beta \in N$, then $(\alpha + \beta)^2$ is equal to

(1) 27 (2) 9

(3) 81 (4) 18

Ans. (3)

Sol. Given that

$$\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta, \alpha, \beta \in N$$

Now,

$$\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \sum_{r=1}^{9}\left(\frac{r}{2^r}\right) \cdot {}^9C_r + \sum_{r=1}^{9}\left(\frac{3}{2^r}\right) \cdot {}^9C_r$$

$$= \sum_{r=1}^{9}\left(\frac{9}{2^r}\right) \cdot {}^8C_{r-1} + 3\sum_{r=1}^{9} {}^9C_r\left(\frac{1}{2}\right)^r \quad \left[\text{Using } \frac{{}^9C_r}{{}^8C_{r-1}} = \frac{9}{r}\right]$$

$$= \frac{9}{2}\sum_{r=1}^{9} {}^8C_{r-1}\left(\frac{1}{2}\right)^{r-1} + 3\left(\sum_{r=0}^{9}\left({}^9C_r\left(\frac{1}{2}\right)^r\right) - 1\right)$$

$$= \frac{9}{2}\left(1+\frac{1}{2}\right)^8 + 3\left(\left(1+\frac{1}{2}\right)^9 - 1\right)$$

$$= \frac{9}{2} \cdot \left(\frac{3}{2}\right)^8 + 3\left(\frac{3}{2}\right)^9 - 3 = 6.\left(\frac{3}{2}\right)^9 - 3$$

Hence, $\alpha = 6, \beta = 3$

Thus $(\alpha + \beta)^2 = 81$

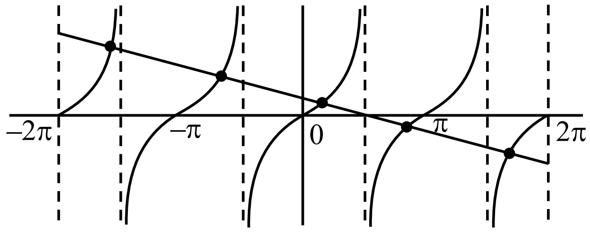
10. The number of solutions of the equation $2x + 3\tan x = \pi$, $x \in [-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$ is

(1) 6 (2) 5

(3) 4 (4) 3

Ans. (2)

Sol. $\tan x = \frac{\pi}{3} - \frac{2x}{3}$



5 solutions

11. If $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$, $x \in \mathbb{R}$, then $\frac{d^2y}{dx^2} + y$ is equal to

(1) –1 (2) 28

(3) 27 (4) 1

Ans. (1)

Sol. $C_3 \to C_3 - C_1$

$$y(x) = \begin{vmatrix} \sin x & \cos x & 1+\cos x \\ 27 & 28 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

$y(x) = -(1 + \cos x)$

$\frac{dy}{dx} = \sin x$

$\frac{d^2y}{dx^2} = \cos x$

$$\boxed{\frac{d^2y}{dx^2} + y = -1}$$

12. Let g be a differentiable function such that $\int_0^x g(t)dt = x - \int_0^x tg(t)dt$, $x \geq 0$ and let $y = y(x)$ satisfy the differential equation $\frac{dy}{dx} - y \tan x = 2(x + 1) \sec x\, g(x)$, $x \in \left[0, \frac{\pi}{2}\right)$. If $y(0) = 0$, then $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $\frac{2\pi}{3\sqrt{3}}$ (2) $\frac{4\pi}{3}$

(3) $\frac{2\pi}{3}$ (4) $\frac{4\pi}{3\sqrt{3}}$

Ans. (2)

Sol. Diff. w.r.t. x

$g(x) = 1 - xg(x)$

$g(x) = \frac{1}{1+x}$

so $\frac{dy}{dx} - y\tan x = 2\sec x$

IF = $e^{-\int \tan dx} = e^{\log\cos x} = \cos x$

solution of D.E.

$y\cos x = \int 2dx + c$

$y\cos x = 2x + c$

$y(0) = 0$

$\boxed{c=0}$

$y = \frac{2x}{\cos x}$

$y = 2x \sec x$

$\boxed{y\left(\frac{\pi}{3}\right) = 2.\frac{\pi}{3}.2 = \frac{4\pi}{3}}$

13. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines

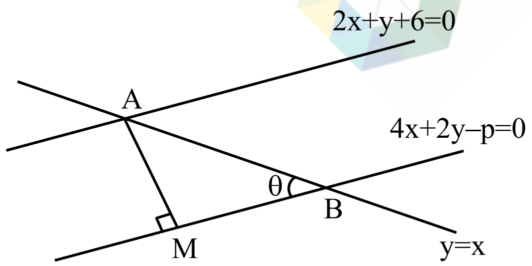
$L_1 : 2x + y + 6 = 0$ and $L_2 : 4x + 2y - p = 0$, $p > 0$, at the points A and B, respectively. If $AB = \frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\frac{AM}{BM}$ is equal to

(1) 5 (2) 4

(3) 2 (4) 3

Ans. (4)

Sol.



Line is $y = x$

$m_1 = 1$, $m_2 = -2$

so $\tan\theta = \left|\frac{1+2}{1-2}\right|$

$\boxed{\tan\theta = \frac{AM}{BM} = 3}$

14. Let $z \in C$ be such that $\frac{z^2+3i}{z-2+i} = 2 + 3i$. Then the sum of all possible values of z^2 is

(1) $19 - 2i$ (2) $-19 - 2i$

(3) $19 + 2i$ (4) $-19 + 2i$

Ans. (2)

Sol. $z^2 + 3i = z(2 + 3i) - 7 - 4i$

$z^2 - z(2 + 3i) + 7 + 7i = 0$ < z_1, z_2

$z_1^2 + z_2^2 = (z_1 + z_2)^2 - 2z_1z_2$

$= 4 - 9 + 12i - 14 - 14i$

$= -19 - 2i$

15. Let $f(x) = \int x^3\sqrt{3-x^2}\,dx$. If $5f(\sqrt{2}) = -4$, then $f(1)$ is equal to

(1) $-\frac{2\sqrt{2}}{5}$ (2) $-\frac{8\sqrt{2}}{5}$

(3) $-\frac{4\sqrt{2}}{5}$ (4) $-\frac{6\sqrt{2}}{5}$

Ans. (4)

Sol. Let $3 - x^2 = t^2$

$+x\,dx = -t\,dt$

$f(x) = \int (3-t^2).t(-t\,dt) + c$

$= \int (t^4 - 3t^2)dt + c$

$= \frac{t^5}{5} - t^3 + c$

$f(x) = \frac{(3-x^2)^{5/2}}{5} - (3-x^2)^{3/2} + c$

$f(\sqrt{2}) = \frac{1}{5} - 1 + c = -\frac{4}{5}$

$\boxed{c=0}$

$f(1) = \frac{2^{5/2}}{5} - 2^{3/2}$

$= 2^{1/2}\left(\frac{4}{5} - 2\right)$

$\boxed{f(1) - \frac{-6\sqrt{2}}{5}}$

16. Let $a_1, a_2, a_3, \ldots$ be a G. P. of increasing positive numbers. If $a_3a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

(1) 131 (2) 130

(3) 129 (4) 128

Ans. **(3)**

Sol. Let the Ist term of G.P. be a & common ratio be r

$a_3a_5 = ar^2.ar^4 = 729$

$= a^2r^6 = 729$

$= ar^3 = 27$ …(i)

$a_2 + a_4 = ar + ar^3 = \frac{111}{4}$

$= ar = \frac{3}{4}$ …(ii)

(i) ÷ (ii)

$\frac{ar^3}{ar} = \frac{27}{3/4}$

$r^2 = 36$

$r = 6$

from (ii)

$a(6) = \frac{3}{4} \Rightarrow a = \frac{1}{8}$

Now, $24(a_1 + a_2 + a_3)$

$= 24(a + ar + ar^2)$

$= 24a(1 + r + r^2)$

$= 24 \times \frac{1}{8}(1 + 6 + 36)$

$= 3(43)$

$= 129$

17. Let the domain of the function

$f(x) = \log_2 \log_4 \log_6 (3 + 4x - x^2)$ be (a, b). If

$\int_0^{b-a}\left[x^2\right] dx = p - \sqrt{q} - \sqrt{r}, p, q, r \in \mathbb{N}$, gcd(p, q, r) = 1,

where [·] is the greatest integer function,

then p + q + r is equal to

(1) 10 (2) 8

(3) 11 (4) 9

Ans. **(1)**

Sol. $\log_4\log_6(3 + 4x - x^2) > 0$

$\log_6(3 + 4x - x^2) > 1$

$3 + 4x - x^2 > 6$

$x^2 - 4x + 3 < 0$

$(x - 1)(x - 3) < 0$

$x \in (1, 3)$

so a = 1 & b = 3

$\Rightarrow \int_0^2 [x^2]dx = ?$

$I = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{\sqrt{4}} [x^2]dx$

$= 0 + |x|_1^{\sqrt{2}} + 2|x|_{\sqrt{2}}^{\sqrt{3}} + 3|x|_{\sqrt{3}}^{\sqrt{4}}$

$= (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$

$= 5 - \sqrt{2} - \sqrt{3} \Rightarrow p + q + r = 10$

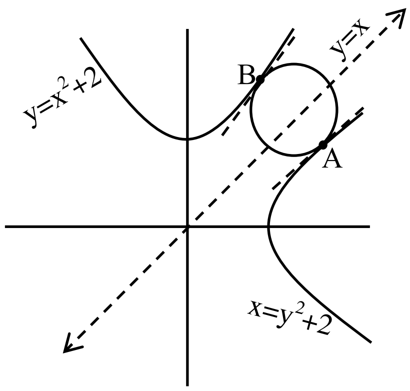
18. The radius of the smallest circle which touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is

(1) $\frac{7\sqrt{2}}{2}$ (2) $\frac{7\sqrt{2}}{16}$

(3) $\frac{7\sqrt{2}}{4}$ (4) $\frac{7\sqrt{2}}{8}$

Ans. **(4)**

Sol. The given parabolas are symmetric about the line y = x.



Tangents at A & B must be parallel to y= x line, so slope of the tangents = 1

$\left(\frac{dy}{dx}\right)_{\min A} = 1 = \left(\frac{dy}{dx}\right)_{\min B}$

For point B, $y = x^2 + 2$

$$\frac{dy}{dx} = 2x = 1$$

$$x = \frac{1}{2} \Rightarrow y = \frac{9}{4}$$

$$\therefore \text{Point B} = \left(\frac{1}{2}, \frac{9}{4}\right) \Rightarrow \text{Point A} = \left(\frac{9}{4}, \frac{1}{2}\right)$$

$$AB = \sqrt{\left(\frac{1}{2} - \frac{9}{4}\right)^2 + \left(\frac{9}{4} - \frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{98}{16}} = \frac{7\sqrt{2}}{4}$$

$$\text{Radius} = = \frac{7\sqrt{2}}{8}$$

19. Let $f(x) = \begin{cases} (1+ax)^{1/x} & , \ x < 0 \\ 1+b & , \ x = 0 \\ \dfrac{(x+4)^{1/2} - 2}{(x+c)^{1/3} - 2} & , \ x > 0 \end{cases}$

be continuous at x = 0. Then $e^a bc$ is equal to

(1) 64 (2) 72

(3) 48 (4) 36

Ans. **(3)**

Sol. $f(0^-) = e^{\lim_{x \to 0} \frac{ax}{x}} = e^a$

$f(0) = 1 + b$

$$f(0^+) = \frac{\frac{1}{2\sqrt{x+4}}}{\frac{1}{3}(x+c)^{-\frac{2}{3}}} = \frac{\frac{1}{2(2)}}{\frac{1}{3}.c^{-\frac{2}{3}}}$$

$$= \frac{3}{4}c^{2/3}$$

Also at x = 0 ;

$c^{1/3} = 2 \Rightarrow c = 8$

So $f(0^+) = \frac{3}{4}(8)^{2/3} = 3$

Now, $e^a = b + 1 = 3$

$e^a.b.c = 3.2.8 = 48$

20. Line L_1 passes through the point (1, 2, 3) and is parallel to z-axis. Line L_2 passes through the point (λ, 5, 6) and is parallel to y-axis. Let for $\lambda = \lambda_1, \lambda_2, \lambda_2 < \lambda_1$, the shortest distance between the two lines be 3. Then the square of the distance of the point $(\lambda_1, \lambda_2, 7)$ from the line L_1 is

(1) 40 (2) 32

(3) 25 (4) 37

Ans. **(3)**

Sol. $L_1 \equiv \frac{x-1}{0} = \frac{y-2}{0} = \frac{z-3}{1}$

$L_2 \equiv \frac{x-\lambda}{0} = \frac{y-5}{1} = \frac{z-6}{0}$

$$SD = \frac{\begin{vmatrix} \lambda-1 & 3 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}$$

$= |\lambda - 1| = 3$

$\lambda = 4, -2$

$\lambda_1 = 4$

$\lambda_2 = -2$

Let foot of perpendicular from P(4, –2, 7) is Q(1, 2, t + 3)

So (3, –4, 4 – t) . (0, 0, 1) = 0

$\boxed{t = 4}$

So Q(1, 2, 7)

$PQ^2 = 9 + 16$

$\boxed{PQ^2 = 25}$

SECTION-B

21. All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by W_n. Let the probability $P(W_n)$ of choosing the word W_n satisfy $P(W_n) = 2P(W_{n-1}), n > 1$.

If $P(CDBEA) = \frac{2^\alpha}{2^\beta - 1}, \alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to : ______

Ans. (183)

Sol. Let $P(W_1) = x$

$$\sum_{i=1}^{120} P(W_i) = 1$$

$$x + 2x + 2^2x + 2^3x + \ldots + 2^{119}x = 1$$

$$\frac{x(2^{120}-1)}{(2-1)} = 1 \Rightarrow x = \frac{1}{2^{120}-1} \quad \ldots(1)$$

Rank of CDBEA

A _ _ _ _ = $\lfloor 4$ = 24

B _ _ _ _ = $\lfloor 4$ = 24

C A _ _ _ _ = $\lfloor 3$ = 6

C B _ _ _ _ = $\lfloor 3$ = 6

C D A _ _ _ = $\lfloor 2$ = 2

C D B A E = 1

C D B E A = 1

64

So, $P(W_{64}) = 2P(W_{63}) = \ldots = 2^{63} P(W_1)$

$$= \frac{2^{63}}{2^{120}-1}$$

$\alpha + \beta = 63 + 120 = 183$

22. Let the product of the focal distances of the point $P(4, 2\sqrt{3})$ on the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be 32. Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then $p^2 + q^2$ is equal to ……

Ans. (120)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \ldots(1)$

$P(4, 2\sqrt{3})$

$PS_1.PS_2 = 32$

$|PS_1 - PS_2| = 2a$

$P(4, 2\sqrt{3})$ lies on H

$\therefore \frac{16}{a^2} - \frac{12}{b^2} = 1$

$16b^2 - 12a^2 = a^2b^2 \quad \ldots(2)$

$|PS_1 - PS_2|^2 = 4a^2$

$PS_1^2 + PS_2^2 - 2PS_1 . PS_2 = 4a^2$

$(ae - 4)^2 + 12 + (ae + 4)^2 + 12 - 64 = 4a^2$

$2a^2e^2 - 8 = 4a^2$

$a^2 + b^2 - 4 = 2a^2$

$b^2 - a^2 = 4$

(2) & (3) $\Rightarrow 16(a^2 + 4) - 12a^2 = a^2(a^2 + 4)$

$\Rightarrow 16a^2 + 64 - 12a^2 = a^4 + 4a^2$

$\Rightarrow a^4 = 64$

$\Rightarrow a^2 = 8$

$\therefore b^2 = 12$

$p^2 + q^2 = 4b^2 + \frac{4b^4}{a^2}$

$= 120$

23. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c}.\hat{d} = 1$, If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d} + \mu\vec{c}|^2$ is equal to _______.

Ans. (5)

Sol. $\vec{a}\times\vec{d}-\vec{b}\times\vec{d}=0$

$(\vec{a}-\vec{b})\times\vec{d}=0$

$\vec{d}=t(\vec{a}-\vec{b})$

$\vec{d}=t(-2\hat{i}-\hat{j}+2\hat{k})$

$|\vec{d}|=1$

$\boxed{|t|=\frac{1}{3}}$

$\vec{c}\,.\,\vec{a}=0$

$\lambda+\mu=0$

$\mu=-\lambda$

$\vec{c}=\lambda(\hat{j}-\hat{k}),\quad \boxed{|\vec{c}|^2=2\lambda^2}$

$\vec{c}\,.\,\hat{d}=1$

$t(-2,-1,2)\,.\,\lambda(0,1,-1)=1$

$\lambda t=\frac{-1}{3}\quad\Rightarrow\boxed{\lambda^2=1}$

$|3\lambda\hat{d}+\mu\vec{c}|^2=9\lambda^2|\hat{d}|^2+\mu^2|\vec{c}|^2+6\lambda\mu(\hat{d}\,.\,\vec{c})$

$=3\lambda^2+2\lambda^4$

$=5$

24. If the number of seven-digit numbers, such that the sum of their digits is even, is $m\cdot n\cdot 10^n$; $m, n\in\{1, 2, 3, \ldots, 9\}$, then $m+n$ is equal to ______

Ans. **(14)**

Sol. Total 7 digit nos. = 9000000

7 digit nos. having sum of digits

Even = 4500000

$=9.5.10^5$

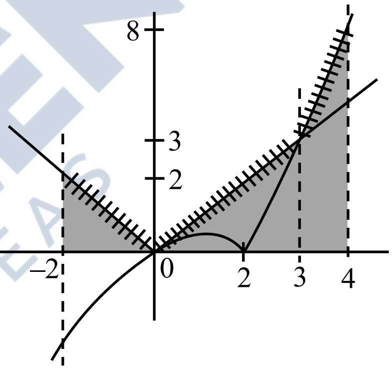
m = 9, n = 5

m + n = 14

25. The area of the region bounded by the curve $y=\max\{|x|, x|x-2|\}$, then x-axis and the lines $x=-2$ and $x=4$ is equal to ______.

Ans. **(12)**

Sol.



Required Area $=\frac{1}{2}\times2\times2+\frac{1}{2}\times3\times3+\frac{1}{2}\times1\times11$

$=12$