JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON FRIDAY 04th APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let a > 0. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1x_2 = 54$, then $a + x_1 + x_2$ is equal to :-

(1) 15

(2) 18

(3) 24

(4) 13

Ans. (2)

Sol. $f'(x) = 18x^2 - 90\,ax + 108\,a^2 = 0$

$x = 2a$ & $x = 3a$

$x_1 = 2a \quad x_2 = 3a$

$x_1x_2 = 54$

$6a^2 = 54$

$a = 3$

$a + x_1 + x_2$

$3 + 2 \times 3 + 3 \times 3 = 18$

option (2)

2. Let f be a differentiable function on **R** such that f(2) 1, $f'(2) = 4$. Let $\lim\limits_{x\to 0}(f(2+x))^{3/x} = e^{\alpha}$. Then the number of times the curve $y = 4x^3 - 4x^2 - 4(\alpha-7)x - \alpha$ meets x-axis is :-

(1) 2 (2) 1

(3) 0 (4) 3

Ans. (1)

Sol. $\lim\limits_{x\to 0}(f(2+x))^{\frac{3}{x}}$

$e^{\lim\limits_{x\to 0}\frac{(f(2+x)-1)3}{x}}$

$e^{3f'(2)} = (e)^{12} = (e)^{a} \Rightarrow a = 12$

$y = 4x^3 - 4x^2 - 4(a-7)x - a$

$y = 4x^3 - 4x^2 - 20x - 12$

roots $x = -1, -1, 3$

option (1)

3. The sum of the infinite series

$$\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cot^{-1}\left(\frac{67}{4}\right)+.....$$

is :-

(1) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right)$ (2) $\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{2}\right)$

(3) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{2}\right)$ (4) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

Ans. (4)

Sol. $T_n = \tan^{-1}\left(\frac{4}{4n^2+3}\right)$

$T_n = \tan^{-1}\left(\frac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$

$T_n = \tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$

$T_1 + T_2 + \dots + T_n = \tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$

$S_\infty = \frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

option (4)

4. Let A ={–3, –2, –1, 0, 1, 2, 3} and R be a relation on A defined by xRy if and only if $2x - y \in \{0, 1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m$ n is equal to :-

(1) 18

(2) 17

(3) 15

(4) 16

Ans. (2)

Sol. $2x - y = 0$

$\{0, 0\}\ \{-1, -2\}\ \{1, 2\}$

$2x - y = 1$

$\{0, -1\}\ \{1, 1\}\ \{2, 3\}\ \{-1, -3\}$

Total (0, 0) (–1, –2), (1,2) (0,-1), (1, 1) (2, 3) (–1, –3)

Reflexive $m = 5$ & $\ell = 7$

Symm. $n = 5$ $\ell + m + n = 17$

option (2)

5. Let the product of $\omega_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $\omega_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ be $\alpha + i\beta$, $i = \sqrt{-1}$. Let p and q be the maximum and the minimum values of $\alpha + \beta$ respectively.

(1) 140 (2) 130

(3) 160 (4) 150

Ans. (2)

Sol. $\omega_1 = (8 \sin\theta + 7 \cos\theta) + i(\sin\theta + 4 \cos\theta)$

$\omega_2 = (\sin\theta + 4 \cos\theta) + i(8 \sin\theta + 7 \cos\theta)$

$\omega_1\omega_2 = 8 \sin^2\theta + 7 \sin\theta \cos\theta + 32 \sin\theta \cos\theta + 28 \cos^2\theta - 8 \sin^2\theta - 32 \sin\theta \cos\theta - 7 \sin\theta \cos\theta - 28 \cos^2\theta + i(\sin^2\theta + 4 \sin\theta \cos\theta + 4 \sin\theta \cos\theta + 16 \cos^2\theta + 64 \sin^2\theta + 56 \sin\theta \cos\theta + 56 \sin\theta \cos\theta + 49 \cos^2\theta)$

$\omega_1\omega_2 = 0 + i(65 \sin^2\theta + 120 \sin\theta \cos\theta + 65 \cos^2\theta)$

$\alpha + \beta = 65 + 60 \sin 2q$

$\alpha + \beta|_{max} = 125$

$\alpha + \beta|_{min} = 5$

Ans. = 125 + 5 = 130

option (2)

6. Let the values of p, for which the shortest distance between the lines $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ and $\vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be a, b, (a < b). Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :-

(1) 9

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) 18

Ans. (3)

Sol. shortest distance $= \frac{|(\bar{a} - \bar{b})| \cdot (\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|}$

where

$\bar{a} = -\hat{i} + 0\hat{j} + 0\hat{k}$

$> \bar{a} - \bar{b} = (-1 - p)\hat{i} - 2\hat{j} - \hat{k}$

$\bar{b} = p\hat{i} + 2\hat{j} + \hat{k}$

$\bar{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\bar{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\frac{1}{16} = \frac{|-1 - p + 4 - 1|}{\sqrt{6}}$

$|-p + 2| = 1$

$p = 3$ & $q = 1$

$\frac{x^2}{1^2} + \frac{y^2}{3^3} = 1$

$L.R = \frac{2a^2}{b} = \frac{2 \times 1}{3} = \frac{2}{3}$

option (3)

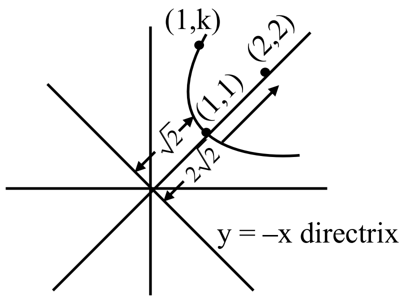
7. The axis of a parabola is the line y = x and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point (1, k) lies on the parabola, then a possible value of k is :-

(1) 4 (2) 9

(3) 3 (4) 8

Ans. **(2)**

Sol.



Directrix x + y = 0

PS = PM

$$\sqrt{(1-2)^2+(K-2)^2}=\frac{(1+K)}{\sqrt{2}}$$

$2K^2 + 8 - 8K + 2 = K^2 + 1 + 2K$

$K^2 - 10K + 9 = 0$

K = 9

option (2)

8. Let the domains of the functions

$f(x) = \log_4\log_3\log_7(8 - \log_2(x^2 + 4x + 5))$ and

$g(x) = \sin^{-1}\left(\frac{7x+10}{x-2}\right)$ be (α, β) and [γ, δ], respectively. Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is equal to :-

(1) 15 (2) 13

(3) 16 (4) 14

Ans. **(1)**

Sol. $\log_3(\log_7(8 - \log_2(x^2 + 4x + 5))) > 0$

$\log_2(x^2 + 4x + 5) < 1$

$x^2 + 4x + 3 < 0$

$\Rightarrow x \in (-3, -1)$

$$-1\le\frac{7x+10}{x-2}\le 1$$

$\Rightarrow x \in [-2, -1]$

$\alpha = -3, \beta = -1, \gamma = -2, \delta = -1$

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 15$

option (1)

9. A line passing through the point A(–2, 0), touches the parabola P : $y^2 = x - 2$ at the point B in the first quadrant. The area, of the region bounded by the line AB, parabola P and the x-axis, is :-

(1) $\frac{7}{3}$ (2) 2

(3) $\frac{8}{3}$ (4) 3

Ans. **(3)**

Sol. Tangent

$y = m(x + 2)$

$y^2 = x - 2$

$(m(n + 2))^2 = n - 2$

$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$

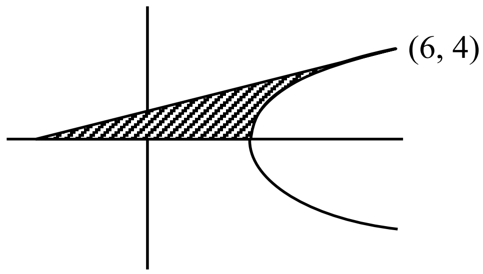
D = 0

$(4m^2 - 1)^2 - 4m^2(4m^2 + 2) = 0$

$m = \frac{1}{4}$

$y = \frac{1}{4}(n+2)$

and point of tangency (6, 2)



$$\text{Area A} = \int_0^2\left((y^2+2)-(4y-2)\right)dy$$

$$A=\frac{8}{3}$$

option (3)

10. Let the sum of the focal distances of the point P(4, 3) on the hyperbola H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H, the length of the latus rectum is l and the product of the focal distances of the point P is m, then $9l^2 + 6$ m is equal to :-

(1) 184 (2) 186

(3) 185 (4) 187

Ans. **(3)**

Sol. $ex + a + ex - a = 8\sqrt{\frac{5}{3}}$

$2ex = 8\sqrt{\frac{5}{3}}$

$2e \times 4 = 8\sqrt{\frac{5}{3}}$

$e = \sqrt{\frac{5}{3}}$

$b^2 = a^2\left(\left(\frac{\sqrt{5}}{3}\right)^2 - 1\right)$

$b^2 = \frac{2}{3}a^2$

$\frac{16}{a^2}-\frac{9}{b^2}=1$

and $b^2 = \frac{2}{3}a^2$

$\Rightarrow a^2 = \frac{5}{2} \quad b^2 = \frac{5}{3}$

Now,

$\ell = \frac{2b^2}{a}$

$\ell^2 = \frac{4b^4}{a^2}$

$9\ell^2 = 36 \times \frac{25}{9\times 5} \times 2$

$9\ell^2 = 40$

$m = (ex + a)(ex - a)$

$m = e^2x^2 - a^2$

$= \frac{5}{3}\times 16 - \frac{5}{2} = \frac{145}{6}$

$= 6m = 145$

$9\ell^2 + 6$ m

$40 + 145 = 185$

option (3)

11. Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 -$ I for $n \geq 3$. Then the sum of all the elements of A^{50} is :-

(1) 53 (2) 52

(3) 39 (4) 44

Ans. **(1)**

Sol. $A^{50} = A^{48} + A^2 - I$

$= A^{46} + 2(A^2 - I)$

$= A^{44} + 3(A^2 - I)$

$= A^2 + 24(A^2 - I)$

$= 25A^2 - 24I$

$= 25\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} - 24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

Sum = 53

option (1)

12. If the sum of the first 20 terms of the series

$\frac{4.1}{4+3.1^2+1^4}+\frac{4.2}{4+3.2^2+2^4}+\frac{4.3}{4+3.3^2+3^4}+\frac{4.4}{4+3.4^2+4^4}+\ldots$

is $\frac{m}{n}$, where m and n are coprime, then m + n is equal to :-

(1) 423 (2) 420

(3) 421 (4) 422

Ans. **(3)**

Sol. $\sum_{r=1}^{20}\frac{4r}{4+3r^2+r^4}$

$\sum_{r=1}^{20}\frac{4r}{(r^2+r+2)(r^2-r+2)}$

$2\sum_{r=1}^{20}\left(\frac{1}{r^2-r+2}-\frac{1}{r^2+r+2}\right)$

$2\left(\frac{1}{2}-\frac{1}{4}\right)$

$\frac{1}{4}-\frac{1}{8}$

$\frac{1}{8}-\frac{1}{14}$

$\left(\frac{1}{382}-\frac{1}{422}\right)$

$=2\left(\frac{1}{2}-\frac{1}{422}\right)$

$=\frac{420}{422}$

$=\frac{210}{211}$

option (3)

13. If

$1^2.\left({}^{15}C_1\right)+2^2.\left({}^{15}C_2\right)+3^2.\left({}^{15}C_3\right)+....+15^2.\left({}^{15}C_{15}\right)$ $= 2^m\cdot3^n\cdot5^k$, where m, n, k $\in$ N, then m + n + k is equal to :-

(1) 19 (2) 21

(3) 18 (4) 20

Ans. **(1)**

Sol. $\sum_{r=1}^{15}r^2\left({}^{15}C_r\right) \Rightarrow 15\sum_{r=1}^{15}r\,{}^{14}C_{r-1}$

$15\sum_{r=1}^{15}(r-1+1)\,{}^{14}C_{r-1}$

$15\cdot14\sum_{r=1}^{15}{}^{13}C_{r-2}+15\sum_{r=1}^{15}{}^{14}C_{r-1}$

$15\cdot14\cdot2^{13} + 15\cdot2^{14}$

$3^1\cdot2^{13}(70 + 10)$

$3^1\cdot2^{13}\cdot80$

$3^1\cdot5^1\cdot2^{17}$

m = 17 n = 1 k = 1

option (1)

14. Let for two distinct values of p the lines y = x + p touch the ellipse E : $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$ at the points A and B. Let the line y = x intersect E at the points C and D. Then the area of the quadrilateral ABCD is equal to

(1) 36 (2) 24

(3) 48 (4) 20

Ans. **(2)**

Sol. Point of contact are $\left(\frac{\mp a^2m}{\sqrt{a^2m^2+b^2}},\frac{\pm b^2}{\sqrt{a^2m^2+b^2}}\right)$

$A\left(\frac{-16}{5},\frac{9}{5}\right)B\left(\frac{16}{5},\frac{-9}{5}\right)$

Point D is $\left(\frac{12}{5},\frac{12}{5}\right)$

Area of ABD = $\frac{1}{2}\begin{vmatrix}-\frac{16}{5} & \frac{9}{5} & 1\\ \frac{16}{5} & \frac{-9}{5} & 1\\ \frac{12}{5} & \frac{12}{5} & 1\end{vmatrix}=12$

Area of ABCD is = 24

option (2)

15. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of AP's in A and B respectively such that D = d + 3, d > 0. If $\frac{p+q}{p-q}=\frac{19}{5}$, then p – q is equal to

(1) 600 (2) 450

(3) 630 (4) 540

Ans. **(4)**

Sol. Let A(a – d, a, a + d) B (b – D, b, b + D)

a = 12 b = 12

p = 12 (144 – d^2)

q = 12 (144 – D^2)

$\frac{p+q}{p-q}=\frac{19}{5}$

$\frac{p}{q} = \frac{24}{14} = \frac{12}{7}$

$\frac{144 - d^2}{144 - (d^2 + 6d + 9)} = \frac{12}{7}$

$1008 - 7d^2 = -12d^2 - 72d + 1620$

$5d^2 + 72d - 612 = 0$

$d = 6$

$D = 9$

$p - q = 12(D^2 - d^3)$

$= 12(81 - 36)$

$= 12(45)$

$= 540$

option (4)

16. If a curve $y = y(x)$ passes through the point $\left(1, \frac{\pi}{2}\right)$ and satisfies the differential equation $(7x^4 \cot y - e^x \operatorname{cosec} y) \frac{dx}{dy} = x^5$, $x \geq 1$, then at $x = 2$, the value of $\cos y$ is:

(1) $\frac{2e^2 - e}{64}$ (2) $\frac{2e^2 + e}{64}$

(3) $\frac{2e^2 - e}{128}$ (4) $\frac{2e^2 + e}{128}$

Ans. **(3)**

Sol. $\frac{dy}{dx} = \frac{7\cot y}{x} - \frac{e^x \operatorname{cosec} y}{x^5}$

$\frac{dy}{dx} = \frac{7\cot y}{\sin y . x} - \frac{e^x}{\sin y x^5}$

$\sin y \frac{dy}{dx} - \cos y . \frac{7}{x} = \frac{-e^x}{x^5}$

let $-\cos y = t$

$\sin y \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + \frac{7t}{x} = \frac{-e^x}{x^5}$

I.F. $= x^7$

$t. x^7 = -\int x^2 e^x dx$

$\cos y\, x^7 = x^2 e^x - 2\int x e^x dx$

$\cos y\, x^7 = x^2 e^x - 2xe^x + 2e^x + c$

$x = 1, y = \frac{\pi}{2}, c = -e$

$\cos y = \frac{2e^2 - e}{128}$

option (3)

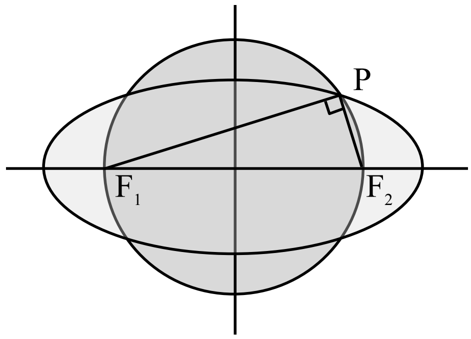
17. The centre of a circle C is at the centre of the ellipse E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$. Let C pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle PF_1F_2 is 30 and the length of the major axis of E is 17, then the distance between the foci of E is :

(1) 26 (2) 13

(3) 12 (4) $\frac{13}{2}$

Ans. **(2)**

Sol.



$\frac{1}{2} PF_1 . PF_2 = 30$

$PF_1 + PF_2 = 17$

$PF_1 = 12$ $PF_2 = 5$

$F_1 F_2 = 13$

option (2)

18. Let $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5$ and $2g(x) - 3g\left(\frac{1}{2}\right) = x,\ x > 0$. If $\alpha = \int_1^2 f(x)\,dx$, and $\beta = \int_1^2 g(x)\,dx$, then the value of $9\alpha + \beta$ is :

(1) 1 (2) 0

(3) 10 (4) 11

Ans. (4)

Sol. $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{1}{x^2} + 5$

$f(x) = \frac{2}{3x^2} - \frac{x^2}{3} + \frac{5}{3}$

$\alpha = \int_1^2 \left(\frac{2}{3x^2} - \frac{x^2}{3} + \frac{5}{3}\right)dx$

$\left(-\frac{2}{3x} - \frac{x^3}{9} + \frac{5x}{3}\right)_1^2$

$-\frac{1}{3} - \frac{8}{9} + \frac{10}{3} + \frac{2}{3} + \frac{1}{9} - \frac{5}{3}$

$\alpha = 2 - \frac{7}{9} = \frac{11}{9}$

$2g(x) - 3g\left(\frac{1}{2}\right) = x$

$g\left(\frac{1}{2}\right) = -\frac{1}{2}$

$g(x) = \frac{x}{2} - \frac{3}{4}$

$\beta = \int_1^2 \left(\frac{x}{2} - \frac{3}{4}\right)dx$

$\left(\frac{x^2}{4} - \frac{3x}{4}\right)_1^2 = 1 - \frac{3}{2} - \frac{1}{4} + \frac{3}{4} = 0$

$9\alpha + \beta = 11$

option (4)

19. Let A be the point of intersection of the lines

$L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}$ and

$L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}$. Let B and C be the point on the lines L_1 and L_2 respectively such that $AB = AC = \sqrt{15}$. Then the square of the area of the triangle ABC is :

(1) 54 (2) 63

(3) 57 (4) 60

Ans. (1)

Sol. Angle between both lines

$\cos\theta = \left|\frac{3+0-5}{\sqrt{2}\sqrt{50}}\right|$

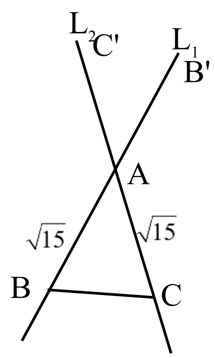
$\sin\theta = \frac{2}{10} = \frac{1}{5}$

$\sin\theta = \frac{\sqrt{24}}{5}$

$\text{area} = \frac{1}{2}ab\sin\theta$

$\frac{1}{2}\sqrt{15}\sqrt{15}\frac{\sqrt{24}}{5}$

square of area $\frac{15 . 15 . 24}{4 . 25}$



option (1)

20. Let the mean and the standard deviation of the observation 2, 3, 3, 4, 5, 7, a, b be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is :

(1) 1 (2) $\frac{3}{4}$

(3) 2 (4) $\frac{1}{2}$

Ans. **(1)**

Sol. $\frac{24+a+b}{8}=4$

$a + b = 8$

$2 = \frac{4+1+1+0+1+9+(a-4)^2+(b-4)^2}{8}$

$16 = 48 + a^2 + b^2 - 8a - 8b$

$a^2 + b^2 = 32$

$32 = 2ab$

$ab = 16$

$a = 4 \quad b = 4$

mode = 4

mean deviation $=\frac{2+1+1+0+1+3+0+0}{8}=1$

option (1)

SECTION-B

21. If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{k=1}^{n}\left(\alpha^k+\frac{1}{\alpha^k}\right)^2 = 20$, then n is equal to _____

Ans. **(11)**

Sol. $\alpha = \omega$

$\therefore \left(\omega^k+\frac{1}{\omega^k}\right)^2 = \omega^{2k}+\frac{1}{\omega^{2k}}+2$

$= \omega^{2k}+\omega^k+2 \qquad \because \omega^{3k}=1$

$\therefore \sum_{k=1}^{n}(\omega^{2k}+\omega^k+2)=20$

$\Rightarrow (\omega^2+\omega^4+\omega^6+\ldots+\omega^{2n})+(\omega+\omega^2+\omega^3+\ldots+\omega^n)+2n=20$

Now if $n = 3m, \quad m \in I$

Then $0 + 0 + 2n = 20 \Rightarrow n = 10$ (not satisfy)

if n = 3m+1, then

$\omega^2 + \omega + 2n = 20$

$-1 + 2n = 20 \Rightarrow n = \frac{21}{2}$ (not possible)

if n = 3m+2,

$(\omega^8 + \omega^{10}) + (\omega^4 + \omega^5) + 2n = 20$

$\Rightarrow (\omega^2 + \omega) + (\omega + \omega^2) + 2n = 20$

$2n = 22$

$n = 11$ satisfy $n = 3m + 2$

$\therefore n = 11$

22. If $\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}dx =$

$\frac{1}{m}\left(\left(\sqrt{1+x^2}+x\right)^n\left(n\sqrt{1+x^2}-x\right)\right) + C$ where C is the constant of integration and m, n $\in$ N, then m + n is equal to

Ans. **(379)**

Sol. rationalise

$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}\times\frac{\left(\sqrt{1+x^2}+x\right)^9}{\left(\sqrt{1+x^2}+x\right)^9}dx$

$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{1}dx$

Put $\sqrt{1+x^2}+x=t$

$\left(\frac{x}{\sqrt{1+x^2}}+1\right)dx = dt$

$dx = \frac{dt}{t}\sqrt{1+x^2}$

Now as $\sqrt{1+x^2} + x = t$

so $\sqrt{1+x^2} - x = \frac{1}{t}$

$\therefore\ \sqrt{1+x^2} = \frac{1}{2}\left(t+\frac{1}{t}\right)$

Thus $I = \int t^{19}.\frac{dt}{t}.\frac{1}{2}\left(t+\frac{1}{t}\right)$

$\Rightarrow \frac{1}{2}\int\left(t^{19}+t^{17}\right)dt$

$= \frac{1}{2}\left(\frac{t^{20}}{20}+\frac{t^{18}}{18}\right)+C$

$= \frac{t^{19}}{360}\left[9t+\frac{10}{t}\right]+C$

$= \frac{t^{19}}{360}\left[9\left(t+\frac{1}{t}\right)+\frac{1}{t}\right]+C$

$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[9\left(2\sqrt{1+x^2}\right)+\left(\sqrt{1+x^2}-x\right)\right]+C$

$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[19\sqrt{1+x^2}-x\right]+C$

$\therefore\ m = 360, n = 19$

$\therefore\ m + n = 379$

23. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, the n is equal to

Ans. (2)

Sol. n cards are drawn & are found all spade, thus

remaining spades = 13 – x

remaining total cards = 52 – x

Now given that P(lost card is spade) = $\frac{11}{50}$

i.e. $\frac{{}^{13-n}C_1}{{}^{52-n}C_1} = \frac{11}{50}$

50(13 – n) = 11(52 – n)

39n = 78

n = 2

24. Let m and n, (m < n) be two 2-digit numbers. Then the total numbers of pairs (m, n), such that gcd (m, n) = 6, is _____

Ans. (64)

Sol. Let m = 6a, n = 6b

$m < n \Rightarrow a < b$

where a & b are co-prime numbers

also since m & n are 2 digit nos, so

$10 \le m \le 99$ & $10 \le n \le 99$

i.e. $2 \le a \le 16$ & $2 \le b \le 16$

($\because$ a is integer)

Now

$2 \le a < b \le 16$ & a & b are co-prime

$\therefore$ if

a = 2, b = 3, 5, 7, 9, 11, 13, 15

a = 3, b = 4, 5, 7, 8, 10, 11, 13, 14, 16

a = 4, b = 5, 7, 9, 11, 13, 15

a = 5, b = 6, 7, 8, 9, 11, 12, 13, 14, 16

a = 6, b = 7, 11, 13

a = 7, b = 8, 9, 10, 11, 12, 13, 15, 16

a = 8, b = 9, 11, 13, 15

a = 9, b = 10, 11, 13, 14, 16

a = 10, b = 11, 13

a = 11, b = 12, 13, 14, 15, 16

a = 12, b = 13

a = 13, b = 14, 15, 16

a = 14, b = 15

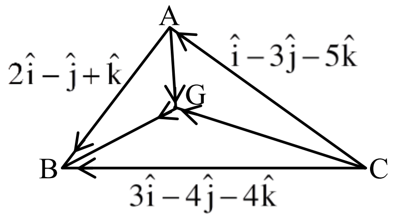
a = 15, b = 16

64 ordered pairs

25. Let the three sides of a triangle ABC be given by the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(\left|\overrightarrow{AG}\right|^2+\left|\overrightarrow{BG}\right|^2+\left|\overrightarrow{CG}\right|^2\right)$ is equal to ____

Ans. (164)

Sol.



By given data

$\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{CB}$

Let pv of $\vec{A}$ are $\vec{O}$ then

$\overrightarrow{AB}=\vec{B}-\vec{A}$

i.e. pv of $\vec{B}- = 2\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{CA}=\vec{A}-\vec{C}$

i.e. pv of $\vec{C}=-\left(\hat{i}-3\hat{j}-5\hat{k}\right)$

Now pv of centroid

$$\left(\vec{G}\right)- = \frac{\vec{A}+\vec{B}+\vec{C}}{3}=\frac{\vec{0}+(2,-1,1)+(-1,3,5)}{3}$$

$$\vec{G}=\frac{1}{3}\left(\hat{i}+2\hat{j}+6\hat{k}\right)$$

Now $\overrightarrow{AG}=\frac{1}{3}\left(\hat{i}+2\hat{j}+6\hat{k}\right)$

$$\Rightarrow \left|\overrightarrow{AG}\right|^2=\frac{1}{9}\times 41$$

$$\overrightarrow{BG}=\left(\frac{1}{3}-2\right)\hat{i}+\left(\frac{2}{3}+1\right)\hat{j}+(2-1)\hat{k}$$

$$\Rightarrow \left|\overrightarrow{BG}\right|^2=\frac{59}{9}$$

$$\overrightarrow{CG}=\left(\frac{1}{3}+1\right)\hat{i}+\left(\frac{2}{3}-3\right)\hat{j}+(2-5)\hat{k}$$

$$\Rightarrow \left|\overrightarrow{CG}\right|^2=\frac{146}{9}$$

Now

$$6\left[\left|\overrightarrow{AG}\right|^2+\left|\overrightarrow{BG}\right|^2+\left|\overrightarrow{CG}\right|^2\right]=6\times\left[\frac{41}{9}+\frac{59}{9}+\frac{146}{9}\right]$$

$$=6\times\frac{246}{9}=164$$