JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON MONDAY 07th APRIL 2025) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. $\lim\limits_{x\to 0^+} \dfrac{\tan\left(5(x)^{\frac{1}{3}}\right)\log_e(1+3x^2)}{\left(\tan^{-1}3\sqrt{x}\right)^2\left(e^{5(x)^{\frac{4}{3}}}-1\right)}$ is equal to

(1) $\frac{1}{15}$ (2) 1

(3) $\frac{1}{3}$ (4) $\frac{5}{3}$

Ans. (3)

Sol.

$$\lim_{x\to 0^+}\left(\frac{\tan\left(5x^{1/3}\right)}{5x^{1/3}}\right)\cdot\left(\frac{\left(3\sqrt{x}\right)^2}{(\tan^{-1}3\sqrt{x})^2}\right)\left(\frac{\ell(1+3x^2)}{3x^2}\right)\left(\frac{5x^{4/3}}{e^{5x^{\frac{4}{3}}}-1}\right)\times\frac{5x^{1/3}.3x^2}{5x^{4/3}.9x}$$

$$=\frac{1}{3}$$

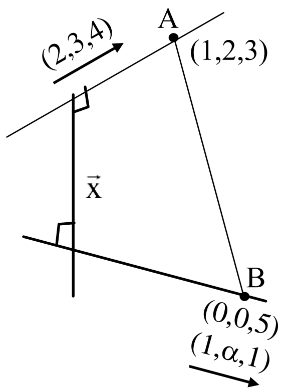
2. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x}{1}=\frac{y}{\alpha}=\frac{z-5}{1}$ is $\frac{5}{\sqrt{6}}$, then the sum of all possible values of α is

(1) $\frac{3}{2}$ (2) $-\frac{3}{2}$

(3) 3 (4) –3

Ans. (4)

Sol.



$L_1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

$L_1 : \frac{x}{1}=\frac{y}{\alpha}=\frac{z-5}{1}$

$$\vec{x}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 1 & \alpha & 1\end{vmatrix}=\hat{i}(3-4\alpha)-\hat{j}(-2)+\hat{k}(2\alpha-3)$$

$$\text{S.D.}=\left|\frac{\overrightarrow{BA}.\vec{n}}{|\vec{n}|}\right|=\left|\frac{(\hat{i}+2\hat{j}-2\hat{k}).\vec{n}}{|\vec{n}|}\right|$$

$\Rightarrow 6(13-8\alpha)^2 = 25((4\alpha-3)^2+(2\alpha-3)^2+16)$

$6(64a^2-280\alpha+169) = 25(20\alpha^2-36\alpha+34)$

$\Rightarrow 116\alpha^2+348\alpha-164=0$

$\alpha_1+\alpha_2=\frac{-348}{116}=-3$

3. Let x = –1 and x = 2 be the critical points of the function $f(x) = x^3 + ax^2 + b\log_e|x| + 1, x \neq 0$. Let m and M respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2,-\frac{1}{2}\right]$. Then $|M + m|$ is equal to

(Take $\log_e 2 = 0.7$):

(1) 21.1 (2) 19.8

(3) 22.1 (4) 20.9

Ans. (1)

Sol. $f(x) = x^2 + ax^2 + b\ell n|x| + 1, \quad x \neq 0$

$f'(x) = 3x^2 + 2ax + \frac{b}{x}$

$f'(-1) = 3 - 2a - b = 0$

$f'(-2) = 12 + 4a - \frac{b}{2} = 0$

$a = \frac{-9}{2}, b = 12$

$f'(x) = 3x^2 - 9x + \frac{12}{x} = \frac{3(x+1)(x+2)^2}{x}$

Max. at n = –1

$f(x) = x^2 - \frac{9}{2}x^2 + 12\ell n|x| + 1$

$f(-1) = -1 - \frac{9}{2} + 1 = -\frac{9}{2}$

$M = -4.5$

Min. value at $x = -2$

$f(-2) = -8 - 18 + 12\ell n2 + 1$

$m = -25 + 12\ell n2 = -16.6$

$|M + m| = 21.1$

4. The remainder when $\left((64)^{(64)}\right)^{(64)}$ is divided by 7 is equal to

(1) 4 (2) 1

(3) 3 (4) 6

Ans. **(2)**

Sol. Let $N = \left((64)^{64}\right)^{64}$

$N = (64)^{64^2}$

$N = (1+63)^{64^2}$, let $64^2 = n$

Expanding by binomial

$N = (1 + 63)^n = 1 + {}^nC_1\, 63 + {}^nC_2(63)^2 + \ldots..$

$= 1 + 63\lambda = 1 + 7\,(9\lambda)$

Remainder when divided by 7 is 1

5. Let P be the parabola, whose focus is (–2, 1) and directrix is $2x + y + 2 = 0$. Then the sum of the ordinates of the points on P, whose abscissa is –2, is

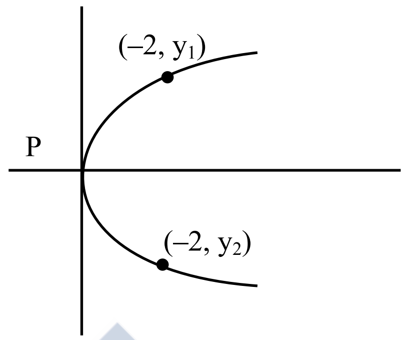
(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) $\frac{1}{4}$ (4) $\frac{3}{4}$

Ans. **(1)**

Sol. Equation of parabola

$$(x + 2)^2 + (y - 1)^2 = \left(\frac{2x+y+2}{\sqrt{5}}\right)^2$$



$5[(x + 2)^2 + (y - 1)^2] = (2x + y + 2)^2$

Put $x = -2$, $5(y - 1)^2 = (y - 2)^2$

$5(y^2 - 2y + 1) = y^2 - 4y + 4$

$\Rightarrow 4y^2 - 6y + 1 = 0 \Rightarrow y_1 + y_2 = \frac{3}{2}$

6. Let $y = y(x)$ be the solution curve of the differential equation $x(x^2 + e^x)dy + (e^x(x - 2)y - x^3)dx = 0$, $x > 0$, passing through the point (1, 0). Then y(2) is equal to :

(1) $\frac{4}{4-e^2}$ (2) $\frac{2}{2+e^2}$

(3) $\frac{2}{2-e^2}$ (4) $\frac{4}{4+e^2}$

Ans. **(4)**

Sol. $x(x^2 + e^x)\, dy + (e^x\,(x - 2)\, y - x^3)\, dx = 0$

$x(x^2 + e^x)\frac{dy}{dx} + e^x(x - 2)\, y = x^3$

$\frac{dy}{dx} + \frac{e^x(x-2)}{x(x^2+e^x)}y = \frac{x^2}{x^2+e^x}$

$\text{I.F.} = e^{\int \frac{e^x(x-2)}{x(x^2+e^x)}dx} = e^{\int \frac{e^x\left(\frac{1}{x^2}-\frac{2}{x^2}\right)dx}{\left(1+\frac{e^x}{x^2}\right)}dx}$

Let $1+\frac{e^x}{x^2}=t \Rightarrow \frac{x^2e^x-e^x2x}{x^4}\,dx=dt$

$\Rightarrow$ I.F. $e^{\ell n\left(1+\frac{e^x}{x^2}\right)}=1+\frac{e^x}{x^2}$

Now $y\left(1+\frac{e^x}{x^2}\right)=\int\frac{x^2}{x^2+e^x}\cdot\frac{x^2+e^x}{x^2}\,dx+C$

$y\left(1+\frac{e^x}{x^2}\right)=x+C$

Passing through (1, 0)

$\Rightarrow C=-1$

$y=\frac{x-1}{1+\frac{e^x}{x^2}}$

$y(2)=\frac{1}{1+\frac{e^2}{4}}=\frac{4}{4+e^2}$

7. From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include atleast 4 batsmen and atleast 4 bowlers. One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is

(1) 165 (2) 155
(3) 145 (4) 135

Ans. (2)

Sol. 7 Batsmen & 6 Bowlers

To select 10 players including atleast

4 Batsmen & 4 Bowlers

Captain & vice-captain already selected

No. of ways $= {}^6C_5\times{}^5C_3+{}^6C_4\times{}^5C_4+{}^6C_3\times{}^5C_5$

$=6\times10+15\times5+20\times1$

$=60+75+20=155$

8. If for $\theta\in\left[-\frac{\pi}{3},0\right]$, the points

$(x,y)=\left(3\tan\left(\theta+\frac{\pi}{3}\right),2\tan\left(\theta+\frac{\pi}{6}\right)\right)$ lie on

$xy+\alpha x+\beta y+\gamma=0$, then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 80 (2) 72
(3) 96 (4) 75

Ans. (4)

Sol. $x=3\left(\frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}\right)$

$x-\sqrt{3}\tan\theta=3\tan\theta+3\sqrt{3}$

$\tan\theta=\frac{x-3\sqrt{3}}{3+\sqrt{3}x}$ ...(1)

$2\left(\frac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\frac{\tan\theta}{\sqrt{3}}}\right)=y$

$2\left(\sqrt{3}\tan\theta+1\right)=y\left(\sqrt{3}-\tan\theta\right)$ ....(2)

using (1) and (2)

$2\left(\frac{x-3\sqrt{3}}{\sqrt{3}+x}+1\right)=y\left(\sqrt{3}-\frac{\left(x-3\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}+x\right)}\right)$

$2\sqrt{3}\left(x-3\sqrt{3}+x+\sqrt{3}\right)=y\left(3\left(\sqrt{3}+x\right)-x+3\sqrt{3}\right)$

$4\sqrt{3}x-12=y\left(2x+6\sqrt{3}\right)$

$xy-2\sqrt{3}x+3\sqrt{3}y-6=0$

$\Rightarrow \alpha=-2\sqrt{3},\ \beta=3\sqrt{3},\ \gamma=-6$

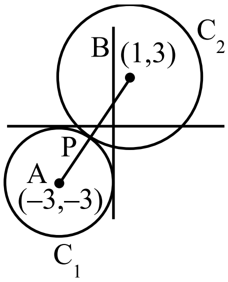
$\alpha^2+\beta^2+\gamma^2=12+27+36=75$

9. Let C_1 be the circle in the third quadrant of radius 3, that touches both coordinate axes. Let C_2 be the circle with centre (1, 3) that touches C_1 externally at the point (α,β). If $(\beta-\alpha)^2=\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to :

(1) 9 (2) 13
(3) 22 (4) 31

Ans. (3)

Sol. $C_1 : (x+3)^2+(y+3)^2=3^2$



Let C_1 and C_2 has centres

A(–3_1 –3) and B(1, 3)

$AB = \sqrt{16+36} = 2\sqrt{13}$

$r_1 = 3$ and $r_2 = 2\sqrt{13} - 3$

$P(\alpha, \beta),\ \alpha = \dfrac{r_1(1) + r_2(-3)}{r_1 + r_2}, \beta = \dfrac{r_1(3) + r_2(-3)}{r_1 + r_2}$

$\alpha = \dfrac{3 - 3\left(2\sqrt{13} - 3\right)}{2\sqrt{13}}, \beta = \dfrac{18 - 6\sqrt{13}}{2\sqrt{13}},$

$(\beta - \alpha)^2 = \left(\dfrac{6}{2\sqrt{13}}\right)^2$

$(\beta - \alpha)^2 = \left(\dfrac{6}{2\sqrt{13}}\right)^2,\quad m + n = 22$

10. The integral $\displaystyle\int_0^{\pi} \frac{(x+3)\sin x}{1+3\cos^2 x}dx$ is equal to :

(1) $\dfrac{\pi}{\sqrt{3}}(\pi+1)$ (2) $\dfrac{\pi}{\sqrt{3}}(\pi+2)$

(3) $\dfrac{\pi}{3\sqrt{3}}(\pi+6)$ (4) $\dfrac{\pi}{2\sqrt{3}}(\pi+4)$

Ans. (3)

Sol. $I = \displaystyle\int_0^{\pi} \frac{(x+3)\sin x}{1+3\cos^2 x}dx$

$I = \displaystyle\int_0^{\pi} \frac{(\pi - x + 3)\sin x}{(1+3\cos^2 x)}dx$

$2I = \displaystyle\int_0^{\pi} \frac{(\pi+6)\sin x.dx}{(1+3\cos^2 x)} = 2\int_0^{\pi/2} \frac{(\pi+6)\sin x}{(1+3\cos^2 x)}$

$I = \displaystyle\int_0^{\pi/2} \frac{(\pi+6)\sin x.dx}{(1+3\cos^2 x)} = \frac{\pi}{3\sqrt{3}}(\pi+6)$

$\sqrt{3}\cos x = t$

$\sqrt{3}\sin x = dt$

11. Among the statements

(S1) **:** The set $\{z \in \mathbb{C} - \{-i\} : |z| = 1$ and $\dfrac{z-i}{z+i}$ is purely real$\}$ contains exactly two elements, and

(S2) **:** The set $\{z \in \mathbb{C} - \{-1\} : |z| = 1$ and $\dfrac{z-1}{z+1}$ is purely imaginary$\}$ contains infinitely many elements.

(1) both are incorrect (2) only (S1) is correct

(3) only (S2) is correct (4) both are correct

Ans. (3)

Sol. $S_1 : |z| = 1,\ \dfrac{z-i}{z+i} = \dfrac{\bar{z}+i}{\bar{z}-i}$

$\Rightarrow (z-i)(\bar{z}-i) = (z+i)(\bar{z}+i)$

$|z|^2 - i(z+\bar{z}) - 1 = |z|^2 + i(z+\bar{z}) - 1$

$i(z+\bar{z}) = 0$

$z+\bar{z} = 2\cos\theta = 0 \Rightarrow \cos\theta = 0$

$z = 0 + 0i,\ |z| \neq 1$

$S_1 : \dfrac{z-1}{z+1} + \dfrac{\bar{z}-1}{\bar{z}+1} = 0$

$(z-1)(\bar{z}+1) + (z+1)(\bar{z}-1) = 0$

$\Rightarrow |z|^2 + (z-\bar{z}) - 1 + |z|^2 + (z-\bar{z}) - 1 = 0$

$|z|^2 = 1$

12. The mean and standard deviation of 100 observations are 40 and 5.1, respectively, By mistake one observation is taken as 50 instead of 40. If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu + \sigma)$ is equal to

(1) 445 (2) 451

(3) 447 (4) 449

Ans. (4)

Sol. Actual means = $\mu = \dfrac{100(40) - 50 + 40}{100}$

$\mu = 40 - \dfrac{1}{10} = 39.9$

Incorrect variance

$(5.1)^2 = \frac{\sum x_i^2}{100} - (\bar{x})^2$

$\sum x_i^2 = 100 \times (40^2) + 100(5.1)^2$

$\sum x_i^2 = 16 \times 10^4 + (5.1)^2 \times 100 = 162601$

$\sigma^2 = \frac{\sum x_i^2 - 50^2 + 40^2}{100} - (\mu)^2$

$\sigma^2 = 1617.01 - (39.9)^2 = 25$

$\sigma = 5$

$10(\mu + \sigma) = 10\,(39.9 + 5)$

$= 10 \times 44.9 = 449$

13. Let x_1, x_2, x_3, x_4 be in a geometric progression. If 2, 7, 9, 5 are subtracted respectively from x_1, x_2, x_3, x_4 then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1\, x_2\, x_3\, x_4)$ is :

(1) 72 (2) 18

(3) 36 (4) 216

Ans. (4)

Sol. $x_1, x_2, x_3, x_4 \rightarrow$ G.P.

Let $a, ar, ar^2, ar^3 \rightarrow$ G.P.

Now $a - 2, ar - 7, ar^2 - 9, ar^3 - 5 \rightarrow$ A.P.

$2(ar - 7) = a - 2 + ar^2 - 9$ …. (i)

$2(ar^2 - 9) = ar - 7 + ar^3 - 5$ … (ii)

Solving $r = 2, a = -3$

$\therefore$ Product = $x_1, x_2, x_3, x_4 = a^4 r^6 = 81 \times 64$

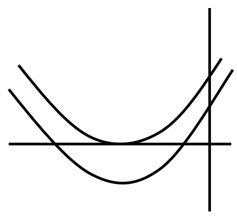
14. Let the set of all values of $p \in \mathbb{R}$, for which both the roots of the equation $x^2 - (p + 2)x + (2p + 9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to

(1) 0 (2) 9

(3) 5 (4) 20

Ans. (3)

Sol. Using location of roots :



(i) $D \geq 0$

(ii) $\frac{-b}{2a} < 0$

(iii) a. $f(0) > 0$

$(p + 2)^2 - 4\,(2p + 9) \geq 0$

$(p + 4)\,(p - 8) \geq 0$ $p + 2 < 0$ $2p + 9 > 0$

Intersection $p \in \left(-\frac{9}{2}, -4\right]$

$\therefore \beta - 2\alpha = -4 + 9 = 5$

15. Let A be a 3×3 matrix such that $|adj\,(adj\,(adj\,A))| = 81$. If

$$S = \left\{ n \in \mathbb{Z} : \left(|adj(adj\ A)|\right)^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)} \right\}$$

, then $\sum_{n \in S} \left|A^{(n^2+n)}\right|$ is equal to

(1) 866 (2) 750

(3) 820 (4) 732

Ans. (4)

Sol. $|adj(adj)(adjA)| = 81$

$\Rightarrow |adjA|^4 = 81$

$\Rightarrow |adjA| = 3$

$\Rightarrow |A|^2 = 3$

$\Rightarrow |A| = \sqrt{3}$

$\left(|A|^4\right)^{\frac{(n-1)^2}{2}} = |A|^{3n^2-5n-4}$

$\Rightarrow 2(n - 1)^2 = 3n^2 - 5n - 4$

$\Rightarrow 2n^2 - 4n + 2 = 3n^2 - 5n - 4$

$\Rightarrow n^2 - n - 6 = 0$

$\Rightarrow (n - 3)(n + 2) = 0$

$\Rightarrow n = 3, -2$

$\sum_{n \in s} |A^{n^2+n}|$

$= |A^2| + |A^{12}|$

$= 3 + 36 = 3 + 729 = 732$

16. If the area of the region bounded by the curves

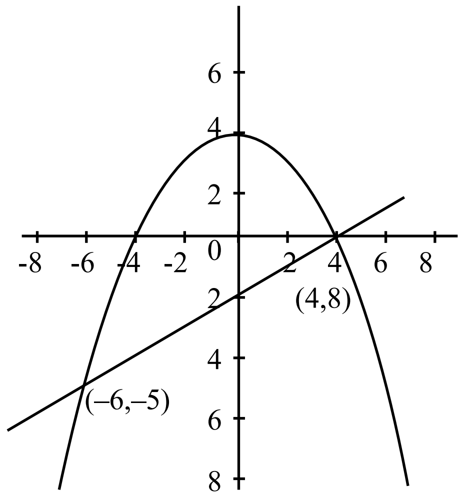
$y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ is equal to α, then 6α equals

(1) 250 (2) 210

(3) 240 (4) 220

Ans. (1)

Sol.



$$\text{Area} = \int_{-6}^{4}\left\{\left(4 - \frac{x^2}{4}\right) - \left(\frac{x-4}{2}\right)\right\}dx$$

$$= \int_{-6}^{4}\left\{-\frac{x^2}{4} - \frac{x-6}{2}\right\}dx$$

$$\alpha = -\frac{x^3}{12} - \frac{x^2}{4} + 6x\Big|_{-6}^{4} = \frac{125}{3}$$

$\therefore 6\alpha = 250$

17. Let the system of equations :

$2x + 3y + 5z = 9,$

$7x + 3y - 2z = 8,$

$12x + 3y - (4 + \lambda)z = 16 - \mu,$

have infinitely many solutions. Then the radius of the circle centred at (λ, μ) and touching the line $4x = 3y$ is

(1) $\frac{17}{5}$ (2) $\frac{7}{5}$

(3) 7 (4) $\frac{21}{5}$

Ans. (2)

Sol. $\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(\lambda+4) \end{vmatrix} = 0$

$\Rightarrow 12(-21) - 3(-39) - (\lambda + 4)(-15) = 0$

$\Rightarrow -252 + 117 + 15(l + 4) = 0$

$\Rightarrow 15\lambda + 177 - 252 = 0$

$\Rightarrow 15\lambda - 75 = 0 \Rightarrow \lambda = 5$

$\begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 7 \\ \mu-8 & 0 & 7 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0$

$\Rightarrow 7 - 7(\mu - 8) = 0 \Rightarrow 1 - (\mu - 8) = 0 \Rightarrow \mu = 9$

$\Rightarrow$ centre of circle (5, 9)

radius = length of ⊥ from centre (5, 9) = $\left|\frac{20-27}{5}\right| = \frac{7}{5}$

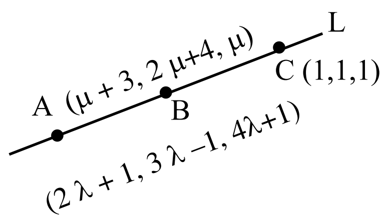
18. Let the line L pass through (1, 1, 1) and intersect the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z}{1}$. Then, which of the following points lies on the line L?

(1) (4, 22, 7) (2) (5, 4, 3)

(3) (10, –29, –50) (4) (7, 15, 13)

Ans. (4)

Sol.



Dr's of AC $\Rightarrow 2\lambda, 3\lambda - 2, 4\lambda$

Dr's of BC $\Rightarrow \mu + 2, 2\mu + 3, \mu - 1$

$\Rightarrow \frac{\mu+2}{2\lambda} = \frac{2\mu+3}{3\lambda-2} = \frac{\mu-1}{4\lambda}$

$\Rightarrow 2(\mu + 2) = \mu - 1 \Rightarrow \mu = -5$

$\Rightarrow$ Dr's of BC $\Rightarrow$ 3,7,6

$\Rightarrow$ equation of L $\Rightarrow \frac{x-1}{3} = \frac{y-1}{7} = \frac{z-1}{6}$

(7, 15, 13) satisfies.

19. Let the angle $\theta, 0<\theta<\frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\vec{c}=3\hat{a}+6\hat{b}+9(\hat{a}\times\hat{b})$, then the value of $9(\vec{c}\cdot\hat{a})-3(\vec{c}\cdot\hat{b})$ is

(1) 31 (2) 27
(3) 29 (4) 24

Ans. **(3)**

Sol. $\vec{c}=3\vec{a}+6\vec{b}+9\left(\vec{a}\times\vec{b}\right)$

$\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)\Rightarrow \sin\theta=\frac{\sqrt{65}}{9}\Rightarrow \cos\theta=\frac{4}{9}$

$\vec{c}.\vec{a}=3|\vec{a}|^2+6\vec{a}.\vec{b}=3+\frac{6.4}{9}=\frac{51}{9}$

$\vec{c}.\vec{a}=3\vec{a}.\vec{b}+6|\vec{b}|^2=\frac{3.4}{9}+6=\frac{22}{3}$

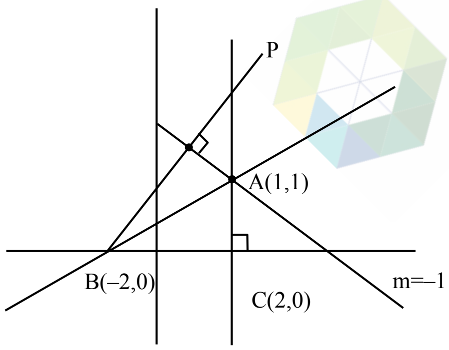
$\therefore\ 9\left(\vec{c}.\vec{a}\right)-3\left(\vec{c}.\vec{b}\right)=51-22=29$

20. Let ABC be the triangle such that the equations of lines AB and AC be 3y – x = 2 and x + y = 2, respectively, and the points B and C lie on x-axis. If P is the orthocentre of the triangle ABC, then the area of the triangle PBC is equal to

(1) 4 (2) 10
(3) 8 (4) 6

Ans. **(4)**

Sol.



Equation of Altitude AP : x = 1
Equation of Altitude BP : y – 0 = 1(x + 2)
$\Rightarrow$ x = 1 &
x – y + 2 = 0
P(1, 3)

Area of $\Delta PBC=\frac{1}{2}\times 4\times 3=6$

SECTION-B

21. The number of points of discontinuity of the function $f(x)=\left[\frac{x^2}{2}\right]-\left[\sqrt{x}\right], x\in[0,4]$, where $[\cdot]$ denotes the greatest integer function is ______

Ans. **(8)**

Sol. Check for $\left[\frac{x^2}{2}\right]$ and $\left[\sqrt{x}\right]$ becomes integers.

$\{0, 1, \sqrt{2}, 2, \sqrt{6}, \sqrt{8}, \sqrt{10}, \sqrt{12}, \sqrt{14}, 4\}$

Continuous at 0^+, continuous at 4^-

$\left[\frac{x^2}{2}\right]=\left[\sqrt{x}\right]$, occurs at $x=\sqrt{2}$

$\Rightarrow$ Not continuous

$\therefore$ function is discontinuous at 8 points.

22. The number of relations on the set A = {1, 2, 3} containing at most 6 elements including (1, 2), which are reflexive and transitive but not symmetric, is ______

NTA Ans. (5)

Allen Ans. (6)

Sol. A = {1,2,3}

(1,1), (2,2), (3,3), (1,2) $\in$ R

Remaining elements are

(2,1), (2,3), (1,3), (3,1), (3,2)

(1) If relation contains exactly 4 elements = 1 way

(2) if relation contains exactly 5 elements

It can be (1, 3), (3, 2) $\Rightarrow$ 2 ways

(3) If relation contain exactly 6 elements

It can be

((2, 3), (1,3)), ((1,3), (3, 2)), ((3,1), (3,2))

$\Rightarrow$ 3 ways.

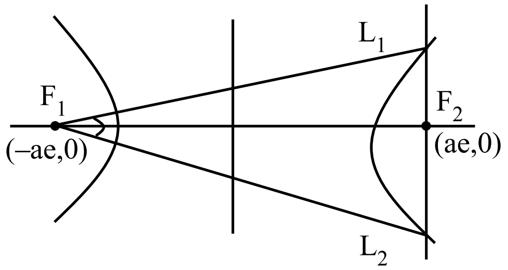
Total = 6 ways

23. Consider the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having one of its focus at P(–3, 0). If the latus ractum through its other focus subtends a right angle at P and $a^2b^2=\alpha\sqrt{2}-\beta, \alpha, \beta \in \mathbb{N}$.

Ans. (1944)

Sol. $f_1 \equiv (-a\,e, 0) \equiv P(-3, 0)$

$\Rightarrow ae = 3$



$$\tan 45^{\circ} = \frac{b^2/a}{2ae}$$

$$2ae = \frac{b^2}{a}$$

$b^2 = 6a$

Also $a^2e^2 = a^2 + b^2$

$9 = a^2 + 6a$

$a^2 + 6a - 9 = 0$

$a = -3 \pm 3\sqrt{2} = -3\left(1 \pm \sqrt{2}\right)$

$\therefore \quad a^2b^2 = a^2 \,.\, 6a = 6a^3$

$= 6(135\sqrt{2} - 189)$

$\alpha = 810$ and $\beta = 1134$

$\therefore \alpha + \beta = 1944$

24. The number of singular matrices of order 2, whose elements are from the set {2, 3, 6, 9} is

Ans. (36)

Sol. $\begin{vmatrix} a & d \\ b & c \end{vmatrix} = ad - bc \Rightarrow ad = bc$

Case-I Exactly 1 no. is used

$\Rightarrow$ All singular = $^4C_1$

Case-II Exactly 2 no. is used

$\Rightarrow {}^4C_2 \times 2 \times 2$

Case-III Exactly 3 no. is used

None will be singular

Case-IV Exactly 4 No. is used

ad = bc

$\Rightarrow 2 \times 9 = 3 \times 6$

$\begin{vmatrix} 9 & - \\ - & 2 \end{vmatrix} \Rightarrow {}^4C_1 \times 21$

Total = 36

25. For $n \geq 2$, let S_n denote the set of all subsets of $\{1, 2.......,n\}$ with **no** two consecutive numbers. For example $\{1, 3, 5\} \in S_6$, but $\{1, 2, 4\} \notin S_6$. Then $n\,(S_5)$ is equal to ______

Ans. (13)

Sol. A = {1, 2, 3, 4, 5......n}

No. of subsets having r elements such that no two are consecutive is = $^{n-r+1}C_r$

for n = 5, no. of ways = $^{6-r}C_r$

Subsets having no element = 1

Subsets having exactly 1 element = $^5C_1 = 5$

Subsets having exactly 2 element = $^4C_2 = 6$

Subsets having exactly 3 element = $^3C_3 = 1$

$\Rightarrow 5 + 6 + 1 + 1 = 13$