JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON FRIDAY 24th JANUARY 2025) **TIME : 9:00 AM TO 12:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{a}.\vec{c} = 5$, then $|\vec{c}|$ is equal to

(1) $\frac{1}{3\sqrt{2}}$ (2) 18

(3) 16 (4) $\sqrt{\frac{11}{6}}$

Ans. (4)

Sol. $\vec{c} = \lambda(\vec{b} \times (\vec{a} \times \vec{b}))$

$= \lambda((\vec{b}.\vec{b})\vec{a} - (\vec{a}.\vec{b})\vec{b})$

$= \lambda(11\vec{a} - 2\vec{b}) = \lambda(11i + 22j + 33k - 6i - 2j + 2k)$

$= \lambda(5i + 20j + 35k)$

$= 5\lambda(5i + 4j + 7k)$

= Given $\vec{c}.\vec{a} = 5$

$= 5\lambda(1 + 8 + 21) = 5 \quad = \lambda = \frac{1}{30}$

$\Rightarrow \vec{c} = \frac{1}{6}(i + 4j + 7k)$

$|\vec{c}| = \frac{\sqrt{1+16+49}}{6} = \sqrt{\frac{11}{6}}$

2. In $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0, then I(9, 14) + I(10, 13) is

(1) I(9, 1) (2) I(19, 27)

(3) I(1, 13) (4) I(9, 13)

Ans. (4)

Sol. $I(m,m) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$

Let $x = \sin^2\theta$ $\quad dx = 2\sin\theta\cos\theta d\theta$

$I(m,n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1}d\theta$

$I(9, 14) + I(10, 13) = 2\int_0^{\pi/2} (\sin\theta)^{17}(\cos\theta)^{27}d\theta$

$+2\int_0^{\pi/2} (\sin\theta)^{19}(\cos\theta)^{25}d\theta$

$= 2\int_0^{\pi/2} (\sin\theta)^{17}(\cos\theta)^{25}d\theta$

= I (9, 13)

3. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function such that $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$. If the $\lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right) = \beta$; $\alpha, \beta \in \mathbb{R}$, then $\alpha + 2\beta$ is equal to

(1) 3 (2) 5

(3) 4 (4) 6

Ans. (3)

Sol. $F(x) - 6f(1/x) = \frac{35}{3x} - \frac{5}{2}$ ……(1)

Replace $x \to \frac{1}{x}$

$F(1/x) - 6(x) = \frac{35x}{3} - \frac{5}{2}$ ……(2)

Using (1) & (2)

$f(x) = -2x - \frac{1}{3x} + \frac{1}{2}$

$B = \lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right)$

$= \lim_{x\to 0}\left(\frac{1}{\alpha x} - 2x - \frac{1}{3x} + \frac{1}{2}\right)$

$\alpha = 3, \quad B = ½$

So, $\alpha + 2B = 3 + 1 = 4$

4. Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term –p and common difference p is $\sqrt{2026 S_{2025}}$, then the absolute difference between 20th and 15th terms of the A.P. is

(1) 25 (2) 90

(3) 20 (4) 45

Ans. (1)

Sol. $Sn = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}$ …. N terms

$S_{2025} = \sum_{n=1}^{2025}\frac{1}{n(n+1)} = \sum_{n=1}^{2025}\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) \ldots\ldots \left(\frac{1}{2025} - \frac{1}{2026}\right)$

$= \frac{2025}{2026}$

$\sqrt{2026.S_{2025}} = \sqrt{2025} = 45$

Given : $\frac{6}{2}[-2p+(6-1)p]=45$

$9p = 45$

$p = 5$

$|A_{20} - A_{15}| = |-5 + 19 \times 5| - [-5 + 14 \times 5]$

$= |90 - 65|$

$= 25$

5. Let $f(x) = \frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. Then the value of $8\left(f\left(\frac{1}{15}\right)+f\left(\frac{2}{15}\right)+...+f\left(\frac{59}{15}\right)\right)$ is equal to

(1) 118
(2) 92
(3) 102
(4) 108

Ans. **(1)**

Sol. $f(x) = \frac{42^x+16}{2.2^{2x}+16.2^x+32}$

$f(x) = \frac{2(2^x+4)}{2^{2x}+8.2^x+16}$

$f(x) = \frac{2}{2^x+4}$

$f(4-x) = \frac{2^x}{2(2^x+4)}$

$f(x) + f(4-x) = \frac{1}{2}$

So, $f\left(\frac{1}{15}\right)+f\left(\frac{59}{15}\right)=\frac{1}{2}$

Similarly $=$ $f\left(\frac{29}{15}\right)+f\left(\frac{31}{15}\right)=\frac{1}{2}$

$f\left(\frac{30}{15}\right)=f(2)=\frac{2}{2^2+4}=\frac{2}{8}=\frac{1}{4}$

$\Rightarrow 8\left(29\times\frac{1}{2}+\frac{1}{4}\right)$

Ans. 118

Option (4)

6. If α and β are the roots of the equation $2z^2 - 3z - 2i = 0$, where $i=\sqrt{-1}$, then $16.\text{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right).\text{Im}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$ is equal to

(1) 398
(2) 312
(3) 409
(4) 441

Ans. **(4)**

Sol. $2z^2 - 3z - 2i = 0$

$2\left(z-\frac{i}{z}\right)=3$

$\alpha-\frac{i}{\alpha}=\frac{3}{2}$

$\Rightarrow \alpha^2-\frac{1}{\alpha^2}-2i=\frac{9}{4}$

$\Rightarrow \alpha^2-\frac{1}{\alpha^2}-2i=\frac{9}{4}$

$\Rightarrow \frac{9}{4}+2i=\alpha^2-\frac{1}{\alpha^2}$

$\Rightarrow \frac{81}{16}-4+9i=\alpha^4+\frac{1}{\alpha^4}-2$

$\Rightarrow \frac{49}{16}+9i=\alpha^4+\frac{1}{\alpha^4}$

Similarly

$\Rightarrow \frac{49}{16}+9i=\beta^4+\frac{1}{\beta^4}$

$\Rightarrow \frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}$

$= \frac{\alpha^{15}\left(\alpha^4+\frac{1}{\alpha^4}\right)+\beta^{15}\left(\beta^4+\frac{1}{\beta^4}\right)}{\alpha^{15}+\beta^{15}}$

$= \frac{(\alpha^{15}+\beta^{15})\left(\frac{49}{16}+9i\right)}{(\alpha^{15}+\beta^{15})}$

Real $= \frac{49}{16}$

Im $= 9$

Ans. 441

7. $\lim_{x\to 0} \text{cosec}x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$ is

(1) 0
(2) $\frac{1}{2\sqrt{5}}$
(3) $\frac{1}{\sqrt{15}}$
(4) $-\frac{1}{2\sqrt{5}}$

Ans. (4)

Sol. $\lim\limits_{x\to 0}\operatorname{cosec}x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$

$$\lim_{x\to 0}\frac{\operatorname{cosec}x\left(\cos^2 x+3\cos x-\sin x-4\right)}{\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{1}{\sin x}\frac{\left(\cos^2 x+3\cos x-4\right)-\sin x}{\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{(\cos x+4)(\cos x-1)-\sin x}{\sin x\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{-2\sin^2\frac{x}{2}(\cos x+4)-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$\lim_{x\to 0}\frac{-\left(\sin\frac{x}{2}(\cos x+4)+\cos\frac{x}{2}\right)}{\cos\frac{x}{2}\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$-\frac{1}{2\sqrt{5}}$$

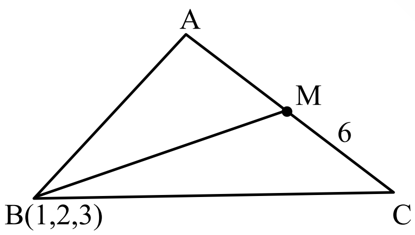
8. Let in a ΔABC, the length of the side AC be 6, the vertex B be (1, 2, 3) and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of ΔABC is

(1) 42 (2) 21

(3) 56 (4) 17

Ans. (2)

Sol.



Let M $(3\lambda+6, 2\lambda+7, -2\lambda+7)$

$\overrightarrow{BM}=(3\lambda+5)\hat{i}+(2\lambda+5)\hat{j}+(-2\lambda+4)\hat{k}$

$\overrightarrow{AC}.\overrightarrow{BM}=0=3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$

$\overrightarrow{BM}=2\hat{i}+3\hat{j}+6\hat{k}$

$|\overrightarrow{BM}|=7$

Area $=\frac{1}{2}\times 6\times 7\ =21$

Option (2)

9. Let $y=y(x)$ be the solution of the differential equation $\left(xy-5x^2\sqrt{1+x^2}\right)dx+(1+x^2)dy=0$, $y(0)=0$. Then $y\left(\sqrt{3}\right)$ is equal to

(1) $\frac{5\sqrt{3}}{2}$ (2) $\sqrt{\frac{14}{3}}$

(3) $2\sqrt{2}$ (4) $\sqrt{\frac{15}{2}}$

Ans. (1)

Sol. $(1+x^2)\frac{dy}{dx}+xy=5x^1\sqrt{1+x^2}$

$$\frac{dy}{dx}+\frac{xy}{1+x^2}=\frac{5x^2}{\sqrt{1+x^2}}$$

$$\therefore \text{I.F.}=e^{\int\frac{x}{1+x^2}dx}=e^{\frac{\ln(1+x^2)}{2}}=\sqrt{1+x^2}$$

$$\therefore\ y\sqrt{1+x^2}=\int\frac{5x^2}{\sqrt{1+x^2}}.\sqrt{1+x^2}dx$$

$$\therefore\ y\sqrt{1+x^2}=\int\frac{5x^2}{\sqrt{1+x^2}}.\sqrt{1+x^2}dx$$

$$y\sqrt{1+x^2}=\frac{5x^3}{3}+C$$

$\because y(0)=0\Rightarrow 0=0+C\Rightarrow C=0$

$$\therefore\ y=\frac{5x^3}{3\sqrt{1+x^2}}$$

$$y\left(\sqrt{3}\right)=\frac{15\sqrt{3}}{32}\ =\boxed{\frac{5\sqrt{3}}{2}}$$

Option (1)

10. Let the product of the focal distances of the point $\left(\sqrt{3},\frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a>b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is

(1) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$ (2) $\frac{1-\sqrt{3}}{\sqrt{2}}$

(3) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$ (4) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

Ans. (3)

Sol. Product of focal distances = $(a + ex_1)(a - ex_1)$

$$= a^2 - e^2x_1^2 = a^2 - e^2(3)$$

$$= a^2 - 3e^2 = \frac{7}{4} \Rightarrow a^2 = \frac{7}{4} + 3e^2$$

$$\Rightarrow 4a^2 = 7 + 12e^2$$

& $\left(\sqrt{3}, \frac{1}{2}\right)$ lines on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\therefore \frac{3}{a^2} + \frac{1}{4b^2} = 1$$

$$\frac{3}{a^2} + \frac{1}{4(a^2)(1-e^2)} = 1$$

$12(1 - e^2) + 1 = 4a^2(1 - e^2)$

$13 - 12e^2 = (7 + 12e^2)(1 - e^2)$

$\Rightarrow 13 - 12e^2 = 7 - 7e^2 + 12e^2 - 12e^4$

$\Rightarrow 12e^4 - 17e^2 + 6 = 0$

$$\therefore e^2 = \frac{17 \pm \sqrt{289 - 288}}{24} = \frac{17 \pm 1}{24} = \frac{3}{4} \,\&\, \frac{2}{3}$$

$$\therefore e = \frac{\sqrt{3}}{2} \,\&\, \sqrt{\frac{2}{3}}$$

$$\therefore \text{difference} = = \frac{\sqrt{3}}{2} - \sqrt{\frac{2}{3}} = \frac{3 - 2\sqrt{2}}{2\sqrt{3}}$$

Option (3)

11. A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8, and B wins if he throws a sum of 8 before A throws a sum of 5. The probability, that A wins if A makes the first throw, is

(1) $\frac{9}{17}$ (2) $\frac{9}{19}$

(3) $\frac{8}{17}$ (4) $\frac{8}{19}$

Ans. **(2)**

Sol. $p(S_5) = \frac{1}{9}$

$p(S_5) = \frac{5}{36}$

$$\text{required prob} = \frac{1}{9} + \frac{8}{9}.\frac{31}{36}.\frac{1}{9} + \left(\frac{8}{9}.\frac{31}{36}\right)^2.\frac{1}{9} + \ldots\infty$$

$$= \frac{\frac{1}{9}}{1 - \frac{62}{81}} = \frac{9}{19}$$

Option(2)

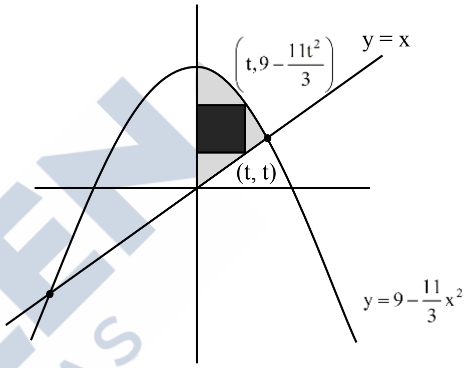
12. Consider the region $R = \left\{(x, y) : x \le y \le 9 - \frac{11}{3}x^2, x \ge 0\right\}$. The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R, is :

(1) $\frac{625}{111}$ (2) $\frac{730}{119}$

(3) $\frac{567}{121}$ (4) $\frac{821}{123}$

Ans. **(3)**

Sol. $t.\left(9 - \frac{11t^2}{3} - t\right)$



$$A = 9t - t^2 - \frac{11}{3}t^3$$

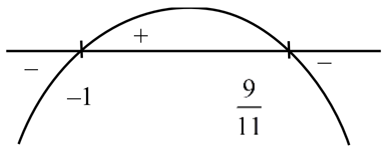
$$\frac{dA}{dt} = 9 - 2t - 11t^2$$

$\Rightarrow 11t^2 + 2t - 9 = 0$

$11t^2 + 11t - 9t - 9 = 0$

$t = -1 \,\&\, t = \frac{9}{11}$

$\therefore \frac{dA}{dt} =$



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Maxima at $t = \frac{9}{11}$

$$\therefore \text{largest area} = \frac{9}{11}\left(9 - \frac{11}{3}, \frac{81}{121} - \frac{9}{11}\right)$$

$$= \frac{9}{11}.\frac{63}{11} = \frac{567}{121}$$

Option (3)

13. The area of the region $\{(x, y) : x^2 + 4x + 2 \le y \le |x + 2|\}$ is equal to

(1) 7 (2) 24/5

(3) 20/3 (4) 5

Ans. **(3)**

Sol. $x^2 + 4x + 2 \le y \le |x + 2|$

The area bounded between

$y = x^2 + 4x + 2 = (x + 2)^2 - 2$

and $y = |x + 2|$ is same as

area bounded between $y = x^2 - 2$ and $y = |x|$

For P.O.I $|x|^2 - 2 = |x|$

$\Rightarrow |x| = 2 \Rightarrow x = \pm 2$

$\therefore$ Required area $= -\int_{-2}^{2}(x^2 - 2)dx + \int_{-2}^{2}|x|\,dx$

$= -2\int_{0}^{2}(x^2 - 2)dx + 2\int_{0}^{2}x.dx$

$= -2\left[\frac{x^3}{3} - 2x\right]_0^2 + 2\left[\frac{x^2}{2}\right]_0^2$

$= --2\left[\frac{8}{3} - 4\right] + 2\left[\frac{4}{2}\right]$

$= -2 \times \left(\frac{-4}{3}\right) + 4$

$= \frac{20}{3}$

14. For a statistical data $x_1, x_2, \ldots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$.

He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is

(1) 7 (2) 4

(3) 9 (4) 5

Ans. **(1)**

Sol. Mean $\bar{x} = 5.5$

$= \sum_{i=1}^{10} x_i = 5.5 \times 10 = 55$

$= \sum_{i=1}^{10} x_i^2 = 371$

$\left(\sum x_i\right)_{new} = 55 - (4+5) + (6+8) = 60$

$\left(\sum x_i\right)_{new} = 371 - (4^2 + 5^2) + (6^2 + 8^2) = 430$

Variance $\sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2$

$\sigma^2 = \frac{430}{10} - \left(\frac{60}{10}\right)^2$

$\sigma^2 = 43 - 36$

$\sigma^2 = 7$

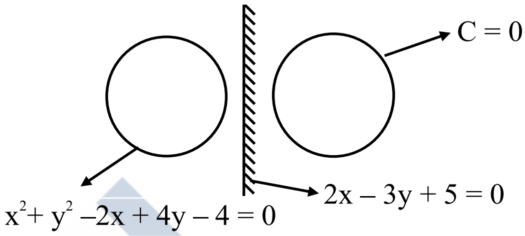
15. Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha,\beta)$, with $\beta < 4$, lies on C such that the length of the arc AB is $(1/6)^{th}$ of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to

(1) 3 (2) $3+\sqrt{3}$

(3) $4-\sqrt{3}$ (4) 4

Ans. **(4)**

Sol.



Centre $(1, -2)$, $r = 3$

Reflection of $(1, -2)$ about $2x - 3y + 5 = 0$

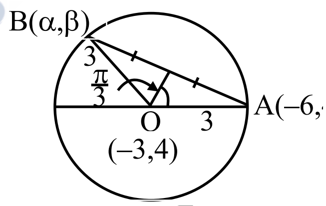
$\frac{x-1}{2} = \frac{y+2}{-3} = \frac{-2(2+6+5)}{13} = -2$

$x = -3, y = 4$

Equation of circle 'C'

$C : (x+3)^2 + (y-4)^2 = 9$

A.T.Q.



$AB = 3\sqrt{3}$

$\ell(\text{arcAB}) = \frac{1}{6} \times 2\pi r$

$r\theta = \frac{1}{6} \times 2\pi r$

$\theta = \frac{\pi}{3}$

$(\alpha + 6)^2 + (\beta - 4)^2 = 27$

$(\alpha + 3)^2 \pm (\beta - 4)^2 = 9$

$(\alpha + 6)^2 - (\alpha + 3)^2 = 18$

$\Rightarrow 6\alpha = -9$

$\Rightarrow \boxed{\alpha = \frac{-3}{2}}, \boxed{\beta = \left(4 - \frac{3\sqrt{3}}{2}\right)}$

$\therefore \beta - \sqrt{3}\alpha$

$\left(4 - \frac{3\sqrt{3}}{2}\right) + \frac{3\sqrt{3}}{2}$

$= 4$

16. For some $n \neq 10$, let the coefficients of the 5^{th}, 6^{th} and 7^{th} terms in the binomial expansion of $(1 + x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1 + x)^{n+4}$ is :

(1) 70 (2) 35

(3) 20 (4) 10

Ans. (2)

Sol. $(1 + x)^{n+4}$

${}^{n+4}C_4, {}^{n+4}C_5, {}^{n+4}C_6, \rightarrow$ A.P.

$\Rightarrow 2 \times {}^{n+4}C_5 = {}^{n+4}C_4 + {}^{n+4}C_6$

$\Rightarrow 4 \times {}^{n+4}C_5 = ({}^{n+4}C_4 + {}^{n+4}C_5) + ({}^{n+4}C_5 + {}^{n+4}C_6)$

$\Rightarrow 4 \times {}^{n+4}C_5 = {}^{n+5}C_5 + {}^{n+5}C_6$

$$\Rightarrow 4 \times \frac{(n+4)!}{5!.(n-1)!} = \frac{(n+6)!}{6!.n!}$$

$$\Rightarrow 4 = \frac{(n+6)(n+5)}{6n}$$

$\Rightarrow n^2 + 11n + 30 = 24n$

$\Rightarrow n^2 - 13n + 30 = 0$

$\Rightarrow n = 3, 10$(rejected)

$\because n \neq 10$

$\therefore$ Largest binomial coefficient in expansion of $(1 + x)^7$

$(\because n + 4 = 7)$

is coeff. of middle term

$\Rightarrow {}^7C_4 = {}^7C_3 = 35$

N.T.A. Ans Option (2)

17. The product of all the rational roots of the equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3$, is equal to :

(1) 14 (2) 7

(3) 28 (4) 21

Ans. (1)

Sol. $(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 3$

Let

$\Rightarrow x^2 - 9x = t$

$\Rightarrow t^2 + 22t + 121 - t - 20 - 3 = 0$

$\Rightarrow t^2 + 21t + 98 = 0$

$\Rightarrow (t + 14)(t + 7) = 0$

$\Rightarrow t = -7, -14$

So, $x^2 - 9x = -7, -14$

$x^2 - 9x + 7 = 0$ or $x^2 - 9x + 14 = 0$

$$x = \frac{9 \pm \sqrt{81-4(7)}}{2 \times 1} \qquad x = \frac{9 \pm \sqrt{81-4(14)}}{2}$$

$$= \frac{9 \pm \sqrt{53}}{2} \qquad = \frac{9 \pm 5}{2}$$

Product of all rational roots $= 7 \times 2 = 14$

Option (1)

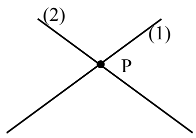
18. Let the line passing through the points $(-1, 2, 1)$ and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ intersect the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ at the point P. Then the distance of P from the point $Q(4, -5, 1)$ is :

(1) 5 (2) 10

(3) $5\sqrt{6}$ (4) $5\sqrt{5}$

Ans. (4)

Sol. Equation of line through point $(-1, 2, 1)$ is $\rightarrow$



$$\Rightarrow \frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} - (2) = \lambda$$

So, $\begin{bmatrix} x = 2\lambda - 1 \\ y = 3\lambda + 2 \\ z = 4\lambda + 1 \end{bmatrix}$

By (1) $\rightarrow \frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu$ (Let)

So, $\begin{bmatrix} x = 3\mu - 2 \\ y = 2\mu + 3 \\ z = \mu + 4 \end{bmatrix}$

For intersection point 'P'

$x = 2\lambda - 1 = 3\mu - 2$

$y = 3\lambda + 2 = 2\mu + 3$ $\begin{bmatrix} \lambda = 1 \\ \mu = 1 \end{bmatrix}$

$z = 4\lambda + 1 = \mu + 4$

So, point $P(x, y, z) = (1, 5, 5)$

& $Q(4, -5, 1)$

$\therefore PQ = \sqrt{9 + 100 + 16}$

$= \sqrt{125} = 5\sqrt{5}$

Option (4)

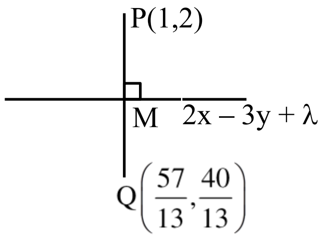
19. Let the lines $3x - 4y - \alpha = 0$, $8x - 11y - 33 = 0$, and $2x - 3y + \lambda = 0$ be concurrent. If the image of the point (1, 2) in the line $2x - 3y + \lambda = 0$ is $\left(\frac{57}{13}, \frac{-40}{13}\right)$, then $|\alpha\lambda|$ is equal to :

(1) 84 (2) 91

(3) 113 (4) 101

Ans. (2)

Sol.



$\because PM = QM$

$$\text{So, } M\left(\frac{\frac{57}{13}+1}{2}, \frac{\frac{-40}{13}+2}{2}\right)$$

$$= \left(\frac{35}{13}, \frac{-7}{13}\right)$$

$\because$ M lies on the time

$2x - 3y + \lambda = 0$

$$2\left(\frac{35}{13}\right) - 3\left(\frac{-7}{13}\right) + \lambda = 0$$

$$\lambda = -\frac{70}{13} + \frac{21}{13}$$

$$= \frac{-91}{13} = -7$$

$$\begin{vmatrix} 3 & -4 & -\alpha \\ 8 & -11 & -33 \\ 2 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 3(-11\lambda - 99) + 4(8\lambda + 66) - \alpha(-24 + 22) = 0$

$\Rightarrow 33\lambda - 297 + 32\lambda + 264 + 24\alpha - 22\alpha = 0$

$\Rightarrow -\lambda + 2\alpha - 33 = 0$ .....(1)

$\therefore \lambda = -7$

$-(-7) + 2\alpha - 33 = 0$

$2\alpha = 26$

$\alpha = 13$

$\therefore |\alpha\lambda| = |13 \times (-7)|$

$= 91$

20. If the system of equations

$2x - y + z = 4$

$5x + \lambda y + 3z = 12$

$100x - 47y + \mu z = 212$,

has infinitely many solutions, then $\mu - 2\lambda$ is equal to

(1) 56 (2) 59

(3) 55 (4) 57

Ans. (4)

Sol. $$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0$$

$2(\lambda\mu + 141) + (5\mu - 300) - 235 - 100\lambda = 0$ ...(1)

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix} = 0$$

$6\lambda = -12 \Rightarrow \lambda = -2$

Put $\lambda = 2$ in (1)

$2(-2\mu + 141) + 5\mu - 300 - 235 + 200 = 0$

$\mu = 53$

$\therefore$ 57

SECTION-B

21. Let f be a differentiable function such that

$$2(x+2)^2 f(x) - 3(x+2)^2 = 10\int_0^x (t+2)f(t)\,dt,$$

$x \geq 0$. Then f(2) is equal to _____.

Ans. (19)

Sol. Differentiate both sides

$4(x+2)f(x) + 2(x+2)^2 f'(x) - 6(x+2) = 10(x+2)f(x)$

$2(x+2)^2 f'(x) - 6(x+2)f(x) = 6(x+2)$

$$(x+2)\frac{dy}{dx} - 3y = 3$$

$$\int \frac{dy}{dx} = 3\int \frac{dx}{x+2}$$

$\ln(y+1) = 3\ln(x+2) + C$

$(y+1) = C(x+2)^3$

$$f(0) = \frac{3}{2}$$

$f(2) = 19$

22. If for some α, β; $\alpha \le \beta$, $\alpha + \beta = 8$ and $\sec^2(\tan^{-1}\alpha) + \text{cosec}^2(\cot^{-1}\beta) = 36$, then $\alpha^2 + \beta$ is ______.

Ans. (14)

Sol. $\text{If}(\tan(\tan^{-1}(\alpha)) + 1\ (\cot(\cot^{-1}\beta))^2 = 36$

$\alpha^2 + \beta^2 = 34$

$\alpha\beta = 15$

$\alpha = 3, \beta = 5$

$\therefore\ \alpha^2 + \beta = 9 + 5 = 14$

23. The number of 3-digit numbers, that are divisible by 2 and 3, but not divisible by 4 and 9, is

Ans. (125)

Sol. No, of 3 digits = 999 – 99 = 900

No. of 3 digit numbers divisible by 2 & 3 i.e. by 6

$\frac{900}{6} = 150$

No. of 3 digit numbers divisible by 4 & 9 i.e. by 36

$\frac{900}{36} = 25$

$\therefore$ No of 3 digit numbers divisible by 2 & 3 but not by 4 & 9

150 – 25 = 125

24. Let be a 3 × 3 matrix such that $X^T AX = O$ for all nonzero 3 × 1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

If $A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and

$\det(\text{adj}(2(A + I))) = 2^\alpha 3^\beta 5^\gamma$, $\alpha, \beta, \gamma, \in N$, then $\alpha^2 + \beta^2 + \gamma^2$ is

Ans. (44)

Sol. $X^T AX = 0$

$$(xyz)\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$(xyz)\begin{pmatrix} a_1x + a_2y + a_3z \\ b_1x + b_2y + b_3z \\ c_1x + c_2y + c_3z \end{pmatrix} = 0$$

$x(a_1x + a_2y + a_3z) + y(b_1x + b_2y + b_3z) + z(c_1x + c_2y + c_3z) = 0$

$a_1 = 0, b_2 = 0\ c_3 = 0$

$a_2 + b_1 = 0, a_3 + c_1 = 0, b_3 = c_2 = 0$

A = skew symm matrix

$$A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\ ;\ A = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$

$x + y = 1$

$-x + z = 4$

$y + z = 5$

$$\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$2x + y = 0 \qquad x = -1$

$-x + z = 4 \qquad y = 2$

$-y - 2z = -8 \qquad z = 3$

$$A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$$

$$2(A+I) = \begin{pmatrix} 2 & -2 & 4 \\ 2 & 2 & 6 \\ -2 & -6 & 2 \end{pmatrix}$$

$2(A+I) = 120 \quad \Rightarrow \det|\text{adi}(2(A+I))|$

$= 120^2 = 2^6.3^2.5^2$

$\alpha = 6, \beta = 2, \gamma = 2$

25. Let $S = \{p_1, p_2......, p_{10}\}$ be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct element of S. Then the number of all ordered pairs (x, y), $x \in S$, $y \in A$, such that x divides y , is ______.

Ans. (5120)

Sol. Let $\frac{y}{x} = \lambda$

$y = \lambda x = 10 \times ({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + ... + {}^9C_9)$

$= 10 \times (2^9)$

10×512

5120