CBSE PAPER - 2022-23 (BASIC) MATHEMATICS

ANSWER AND SOLUTIONS

SECTION-A

1. Option (3)

$2^5 \times 3^2$

2. Option (2)

$60°$

3. Option (4)

$\frac{1}{2}$

4. Option (2)

49

5. Option (4)

$3\sqrt{2}$ units

6. Option (2)

4

7. Option (1)

0

8. Option (1)

4 : 7

9. Option (3)

$\frac{6}{5}$ cm

10. Option (2)

–5, 6

11. Option (2)

1

12. Option (4)

$30°$

13. Option (4)

πr^3

14. Option (1)

7 cm

15. Option (3)

SAS (Side – Angle – Side) Similarity

16. Option (2)

$99°$

17. Option (1)

30 cm

18. Option (3)

24 cm

19. Option (2)

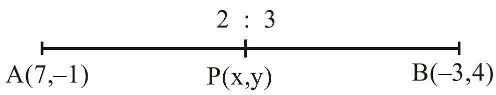
Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Option (1)

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION-B

21. (a)



$$x = \frac{2\times(-3)+3\times 7}{5} = \frac{15}{5} = 3$$

$$y = \frac{2\times 4+3\times(-1)}{5} = \frac{5}{5} = 1$$

coordinate of P are (3, 1)

OR

(b) AB = 10 unit = $AB^2 = 100$

$\Rightarrow (11-3)^2 + (y+1)^2 = 100$

$\Rightarrow y + 1 = \pm 6$

y = 5, –7

22. $\tan^2 60° - 2\ \text{cosec}^2 30° - 2\tan^2 30°$

$$= \left(\sqrt{3}\right)^2 - 2(2)^2 - 2\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 - 8 - \frac{2}{3}$$

$$= \frac{-15-2}{3} = \frac{-17}{3}$$

23. $92 = 2 \times 2 \times 23$

$510 = 2 \times 3 \times 5 \times 17$

HCF = 2

LCM = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

24. (a) x + y = 6 ....(1)

2x – 3y = 4 ....(2)

[Eq.(1)] × 2

$$\begin{aligned} \cancel{2x} + 2y &= 12 \\ \cancel{2x} - 3y &= 4 \\ - \quad + \quad &\quad - \\ \hline 5y &= 8 \end{aligned}$$

$$\Rightarrow y = \frac{8}{5}$$

From (1)

$$x + \frac{8}{5} = 6$$

$$x = 6 - \frac{8}{5} = \frac{30-8}{5} = \frac{22}{5}$$

$$x = \frac{22}{5}, y = \frac{8}{5}$$

OR

(b) 5x – 3y = 11, –10x + 6y = 22

$$\frac{5}{-10} = \frac{-3}{6} \neq \frac{-11}{-22}$$

$$\text{or } -\frac{1}{2} = -\frac{1}{2} \neq \frac{1}{2}$$

$\Rightarrow$ given pair of linear equations is inconsistent.

25. In ΔABC and ΔAMP

$\angle ABC = \angle AMP$ (each 90°)

$\angle BAC = \angle MAP$ (common)

$\Delta ABC \sim \Delta AMP$ (By AA similarity)

SECTION-C

26. (a) LHS = $\sec\theta\,(1-\sin\theta)(\sec\theta + \tan\theta)$

$$= \frac{1}{\cos\theta}(1-\sin\theta)\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{\cos\theta}(1-\sin\theta)\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1 = \text{RHS.}$$

Hence proved

OR

(b) LHS = $\dfrac{1+\sec\theta}{\sec\theta} = \dfrac{1+\dfrac{1}{\cos\theta}}{\dfrac{1}{\cos\theta}} = \dfrac{\dfrac{1+\cos\theta}{\cos\theta}}{\dfrac{1}{\cos\theta}}$

= 1 + cosθ

$$= \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)} = \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta} = \text{RHS}$$

Hence proved

27. $AB = \sqrt{(4-1)^2 + (2-7)^2} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{34}$

$CD = \sqrt{(-4+1)^2 + (4+1)^2} = \sqrt{34}$

DA = $\sqrt{(-4-1)^2+(4-7)^2}=\sqrt{34}$

$\therefore$ AB = BC = CD = DA

AC = $\sqrt{(1+1)^2+(7+1)^2}=\sqrt{68}$

BD = $\sqrt{(4+4)^2+(2-4)^2}=\sqrt{68}$

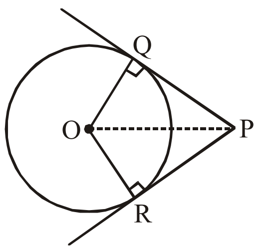
$\therefore$ AC = BD

Hence ABCD is a square.

28. Given : A circle with centre O and PQ, PR are tangents to circle from an external point P.

To prove : PQ = PR

Construction : Join OP, OQ, OR



Proof : In ΔOPQ and ΔOPR

OP = OP (common)

OQ = OR (Radii of the same circle)

∠OQP = ∠ORP (each 90°)

⇒ ΔPOQ ≅ ΔPOR (RHS congruence)

$\therefore$ PQ = PR (By cpct)

Hence Proved

29. $P(x) = x^2 + 3x + 2$

let α, β are its zeros

⇒ α + β = –3, αβ = 2

Now (α + 1) + (β + 1) = α + β + 2 = –3 + 2 = –1

(α + 1)(β + 1) = αβ + (α + β) + 1

= 2 – 3 + 1 = 0

$\therefore$ Required polynomial is

$k(x^2 + x)$ or $x^2 + x$

30. Let $3 + 7\sqrt{2}$ is a rational number

$\Rightarrow 3 + 7\sqrt{2} = \frac{p}{q}$, p, q are integers $q \neq 0$

$\Rightarrow 3 + 7\sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p-3q}{7q}$

RHS is rational but LHS is irrational

$\therefore$ Our assumption is wrong.

Hence $3 + 7\sqrt{2}$ is an irrational number.

31. (a) In ΔABE, DF||AE (given)

$\frac{BD}{DA} = \frac{BF}{FE}$ ... (i) (By BPT)

In ΔABC, DE || AC

$\frac{BD}{DA} = \frac{BE}{EC}$ ... (ii) (By BPT)

from (i) & (ii)

$\frac{BF}{FE} = \frac{BE}{EC}$

Hence proved

OR

(b) In ΔAOB and ΔCOD

$\frac{AO}{OB} = \frac{CO}{OD} \Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

∠AOB = ∠COD (vertically opp.∠s)

⇒ ΔAOB ~ ΔCOD (SAS similarity)

⇒ ∠CAB = ∠ACD (or ∠DBA = ∠BDC)

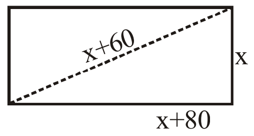
But these are alt. int. ∠s

$\therefore$ AB || CD

⇒ ABCD is trapezium

SECTION-D

32. (a)



There is an error in question diagonal of rectangular field is longest given situation (Not Possible)

OR

(b) Let age of Father = x years

age of son = (45–x) years

Five years ago, age of father = (x – 5) years

Age of son = (40 – x)

ATQ, (x – 5) (40 – x) = 124

$x^2 - 45x + 324 = 0$

(x – 36) (x – 9) = 0

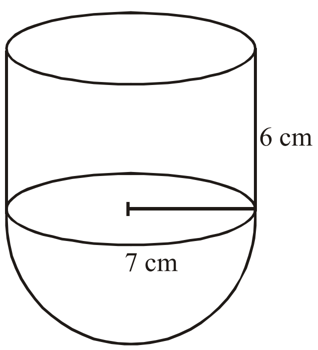
x = 36, x = 9 (Rejected)

∴ Father's age = 36 years & Son's age = 9 years

33. Radius of Hemispherical Bowl = Radius of Cylinder

= 7 cm

Height of Cylinder = 13 – 7 = 6 cm



inner surface area of vessel

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r) = 2 \times \frac{22}{7} \times 7\ (6+7)$

$= 44 \times 13 = 572\ cm^2$

vol. of vessel $= \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2 \left(h + \frac{2}{3}r\right) = \frac{22}{7} \times 7 \times 7 \times \left(6 + \frac{14}{3}\right)$

$= \frac{4928}{3} cm^3 = 1642.67 cm^3$

34.

Daily Exp.(Rs.)	No. of House hold	x_i	$f_i x_i$
100 – 150	4	125	500
150 – 200	5	175	875
200 – 250	12	225	2700
250 – 300	2	275	550
300 – 350	2	325	650
	= 25		= 5275

mean $= \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{5275}{25} = 211$

Mode : modal class = 200 – 250

$\ell = 200$

$f_1 = 12$

$f_0 = 5$

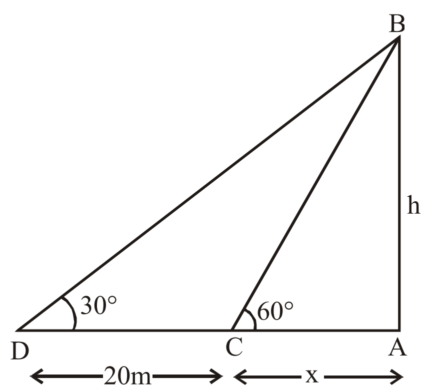
$f_2 = 2$

Mode $= \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 200 + \left(\frac{12-5}{24-5-2}\right) \times 50$

$= \frac{3750}{17} = 220.59\,(\text{approx})$

35. (a) In Δ ABC



In ΔABC

$\tan 60° = \frac{h}{x} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}x$ ....(1)

In ΔABD

$\tan 30° = \frac{h}{x+20} = \frac{1}{\sqrt{3}}$

$\Rightarrow \sqrt{3}h = x + 20$

$\sqrt{3}(\sqrt{3}x) = x + 20$ from eq. (1)

$\Rightarrow 3x = x + 20$

$\Rightarrow 2x = 20$

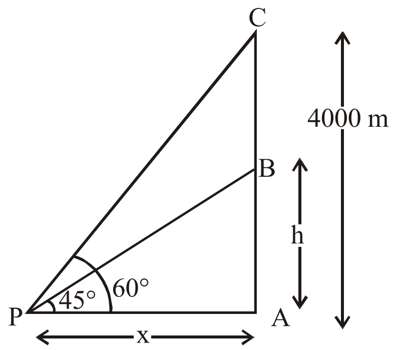
$x = 10$ m

$h = \sqrt{3}x = 10\sqrt{3}m$

$\therefore$ height of tower $10\sqrt{3}$ m **or** 17.3 m

OR

(b)



In $\Delta APB, \frac{h}{x} = \tan 45°$

$\frac{h}{x} = 1$

$\Rightarrow h = x$ ....(1)

In ΔAPC $\frac{4000}{x} = \tan 60° = \sqrt{3}$

$\Rightarrow h = x = \frac{4000}{\sqrt{3}}$

Distance between aeroplanes $= 4000 - \frac{4000}{\sqrt{3}}$

$= 4000\left(1 - \frac{1}{\sqrt{3}}\right)$

$= \frac{5072}{3}$m $= 1690.66$ m (approx)

SECTION-E

36. $a = 2, d = 3$

(i) No. of pots in 10^{th} Row

$= a_{10} = a + 9d = 2 + 9(3) = 29$

(ii) $a_5 - a_2 = (a + 4d) - (a + d) = 3d$

$= 3(3) = 9$

(iii) $S_n = 100 = \frac{n}{2}[2(2) + (n-1)3]$

$\Rightarrow 3n^2 + n - 200 = 0$

$(3n + 25)(n - 8) = 0$

$\therefore n = 8, \quad n = -\frac{25}{3}$ (Reject)

OR

(iii) $S_{12} = \frac{12}{2}(2(2) + 11(3)) = 222$

37. (i) Area of square ABCD $= (40 \text{ cm})^2$

$= 1600 \text{ cm}^2$

(ii) Area of circle $= \pi r^2 = \frac{22}{7} \times 10 \times 10$

$= \frac{2200}{7}\text{cm}^2 = 314.28\text{m}^2$

(iii) Area of 4 quadrants $= 4\left(\frac{1}{4}\pi r^2\right)$

$= \frac{2200}{7}\text{cm}^2$

Remaining area $= 1600 - \left(\frac{2200}{7} + \frac{2200}{7}\right)$

$= 1600 - \frac{4400}{7} = \frac{6800}{7}\text{cm}^2 = 971.43 \text{ cm}^2$

OR

(iii) Area of 4 quadrant $= 4\left(\frac{1}{4}\pi r^2\right) = \frac{2200}{7}\text{cm}^2$

combined area of circle + 4 quadrant

$= \frac{2200}{7} + \frac{2200}{7} = \frac{4400}{7} = 628.57\text{cm}^2$

38. (i) Probability (Type O) = $\frac{21}{50}$

(ii) No. of people with AB Type blood group

= 50 – (21 + 22 + 5) = 2

Probability (Type AB) = $\frac{2}{50} = \frac{1}{25}$

(iii) Probability (Neither Type A nor Type B)

$= \frac{21+2}{50} = \frac{23}{50}$

OR

(iii) Probability (Type A or Type B or Type O)

$= \frac{21+22+5}{50} = \frac{24}{25}$

MATHEMATICS SAMPLE PAPER # 1

ANSWER AND SOLUTIONS

SECTION-A

1. Option (2)

42

2. Option (1)

2 Mean = 3 Median – Mode

3. Option (3)

$2x^2 - 7x + 6 = 0$

4. Option (2)

$5^2 \times 13$

5. Option (1)

$\frac{1}{26}$

6. Option (3)

$\angle B = \angle D$

7. Option (3)

5.0100100001....

8. Option (3)

3

9. Option (1)

25°

10. Option (2)

(– 3, 5)

11. Option (3)

(2, 3)

12. Option (4)

2

13. Option (1)

1

14. Option (3)

0

15. Option (2)

2 cm

16. Option (1)

38.5 cm^2

17. Option (2)

14 cm

18. Option (4)

$\frac{1}{2}$

19. Option (2)

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Option (1)

Assertion (A) is true and Reason is true and reason is the correct explanation of assertion.

SECTION-B

21. Outcomes are {HH, TT, HT, TH}

Favourable outcome {HH}

P (Two Head) = $\frac{1}{4}$

22. Good bulbs = 25 – 5 = 20

P (good bulb) = $\frac{20}{25} = \frac{4}{5}$

OR

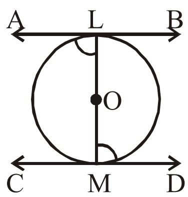
Of all those outcomes, the one's for which a + b = 8 are

2 + 6, 3 + 5, 4 + 4, 5 + 3, 6 + 2 or 5 outcomes.

Probability of getting sum 8

P = 5/36

23.



AB and CD are tangents to a circle with centre O

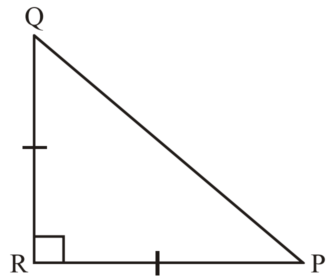
$\angle OLA = 90°$

$\angle OMD = 90°$

$\angle OLA = \angle OMD$

which are alternate angles, hence, AB || CD

24. $\angle R = 90°$



PR = QR [ΔPQR is isosceles Δ]

$\angle P = 45° = \angle Q$

$\sin P = \sin 45° = \frac{1}{\sqrt{2}}$

OR

$\cot A = \frac{8}{15}$

$\text{cosec}A = \sqrt{1+\cot^2 A} = \sqrt{1+\left(\frac{8}{15}\right)^2}$

$= \sqrt{1+\frac{64}{225}}$

$= \sqrt{\frac{289}{225}}$

$= \frac{17}{15}$

25. For equal roots

D = 0

$b^2 - 4ac = 0$

$4 - 4k = 0$

$k = 1$

SECTION-C

26. $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x - 5) + 2(x - 5) = 0$

$(x - 5)(x + 2) = 0$

$x = 5, -2$

Sum of the roots $= \frac{-b}{a} = \frac{3}{1}$

which is same as $5 - 2 = 3$

product of the roots $= \frac{c}{a} = -10$

which is same as $5 \times (-2) = -10$

Hence verified

27. Area of track $= 120 \times 7 \times 2 + \pi(35)^2 - \pi(28)^2$

$= 120 \times 14 + \frac{22}{7}\,[(35)^2 - (28)^2]$

$= 1680 + \frac{22}{7} \times 7 \times 63$

$= 1680 + 1386$

$= 3066\ m^2$

No, Meena is wrong.

28. $\text{L.H.S.} = \frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$

$= \frac{\cos A(\frac{1}{\sin A} - 1)}{\cos A(\frac{1}{\sin A} + 1)} = \frac{(\frac{1}{\sin A} - 1)}{\frac{1}{\sin A} + 1}$

$= \frac{\text{cosec}A - 1}{\text{cosec}A + 1} = \text{R.H.S}$

OR

$\text{L.H.S.} = \frac{\tan A + \sin A}{\tan A - \sin A}$

$= \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A} = \frac{\sin A\,[\sec A + 1]}{\sin A\,[\sec A - 1]} = \frac{\sec A + 1}{\sec A - 1}$

$= \text{R.H.S}$

29. Let us assume, to the contrary, that $\sqrt{3}$ is rational.

So, we can find coprime integers a and b ($\neq 0$) such that

$\sqrt{3} = \frac{a}{b}$, $b \neq 0$, a, $b \in I$

$\Rightarrow \sqrt{3}\,b = a$

Squaring on both sides, we get

$3b^2 = a^2$

Therefore, 3 divides a^2.

(by fundamental theorem of arithmetic)

Therefore, 3 divides a

So, we can write a = 3c for some integer c.

Substituting for a, we get

$3b^2 = 9c^2$

$\Rightarrow b^2 = 3c^2$

This means that 3 divides b^2, and so 3 divides b.

Therefore, a and b have at least 3 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradict our assumption that $\sqrt{3}$ is rational.

So, we conclude that $\sqrt{3}$ is irrational.

OR

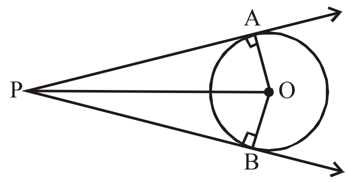
Maximum number of columns = HCF(616, 32)

$616 = 2^3 \times 7 \times 11$

$32 = 2^5$

HCF = 2^3 = 8

30.



In ΔOPA and ΔOPB

$\angle PAO = \angle PBO$ (each 90°)

OP = OP (common)

OA = OB (radii of same circle)

$\Delta OPA \cong \Delta OPB$ (by RHS congruency axiom)

Hence PA = PB (CPCT)

31. $2x + 3y = 11$ .....(1)

$x - 2y = -12$

$x = 2y - 12$ .....(2)

Substitute value of x from (2) in (1), we get

$2(2y - 12) + 3y = 11$

$\Rightarrow 4y - 24 + 3y = 11$

$\Rightarrow 7y = 35$

$\Rightarrow y = 5$

Substituting value of y = 5 in equation (3), we get

$x = 2(5) - 12 = 10 - 12 = -2$

Hence x = –2, y = 5 is the required solution

Now, $5 = -2m + 3$

$\Rightarrow 2m = 3 - 5$

$\Rightarrow 2m = -2$

$\Rightarrow m = -1$

SECTION-D

32. Let common difference be d

$\Rightarrow \frac{14}{2}[2(10) + (14 - 1)d] = 1050$

$\Rightarrow d = 10$

$a_{20} = a + 19d$

$= 10 + 19(10) = 200$

$S_{20} = \frac{20}{2}(10 + 200) = 2100$

OR

a = 5

$a_n = 45$

$S_n = 400$

$\Rightarrow \frac{n}{2}(5 + 45) = 400$

50n = 800

n = 16

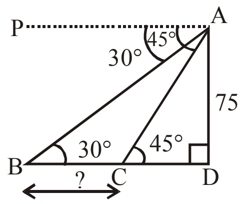
also $a_n = 45$

5 + 15d = 45

15d = 40

d = 8/3

33.



Let AD be the light house and C and B be two ships

In ΔADC, $\tan 45° = \dfrac{75}{CD}$

$1 = \dfrac{75}{CD} \Rightarrow CD = 75$

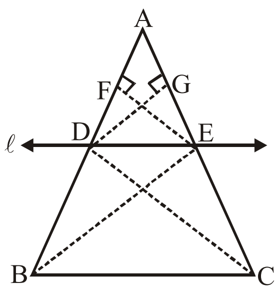
In ΔADB, $\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{75}{BD}$

$\Rightarrow BD = 75\sqrt{3}$

$\Rightarrow$ Distance between two ships = BC = $75(\sqrt{3}-1)$m = 54.9 m

34. **Given :** A ΔABC in which line ℓ parallel to BC (DE||BC) intersecting AB at D and AC at E.

To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



Construction : Join D to C and E to B. Through E draw EF perpendicular to AB i.e., EF $\perp$ AB and through D draw DG $\perp$ AC.

Proof :

Area of (ΔADE) = $\dfrac{1}{2}$(AD × EF) ...(1)

(Area of Δ = $\dfrac{1}{2}$ base × altitude)

Area of (ΔBDE) = $\dfrac{1}{2}$(BD × EF) ...(2)

Dividing (1) by (2)

$$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta BDE)} = \frac{\frac{1}{2}AD \times EF}{\frac{1}{2}BD \times EF} = \frac{AD}{DB} \quad ...(3)$$

Similarly, $\dfrac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta CDE)} = \dfrac{\frac{1}{2}AE \times DG}{\frac{1}{2}EC \times DG} = \dfrac{AE}{EC}$

$$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta CDE)} = \frac{AE}{EC} \quad ...(4)$$

Area (ΔBDE) = Area (ΔCDE) ...(5)

[Δs BDE and CDE are on the same base DE and between the same parallel lines DE and BC.]

From (4) and (5)

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta BDE)} = \frac{AE}{EC} \quad ...(6)$$

From (3) and (6)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

35. Area of shaded region

$\text{ar}(\overset{\frown}{APD}) - \text{ar}(\overset{\frown}{AQB}) - \text{ar}(\overset{\frown}{CSD}) + \text{ar}(\overset{\frown}{BRC})$

$$= \left(\frac{1}{2}\times\pi\times7\times7 - 2\times\frac{1}{2}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\right)$$

$$+\frac{1}{2}\times\pi\times\frac{7}{2}\times\frac{7}{2}$$

$$= \pi\left[\frac{49}{2} - \frac{1225}{400}\right] + \frac{49}{8}\pi$$

$$= \pi\left[\frac{49}{2} - \frac{49}{16}\right] + \frac{49}{8}\pi$$

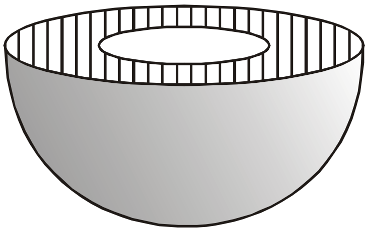
$$= \pi\left[\frac{49}{2} - \frac{49}{16} + \frac{49}{8}\right]$$

$$= \frac{441}{16}\times\frac{22}{7}\text{cm}^2$$

$$= \frac{9702}{112}\text{cm}^2 = 86.625\,\text{cm}^2$$

OR

R = 8 cm, r = 6 cm



Total surface area = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= \pi[8^2 \times 2 + 6^2 \times 2 + (8^2 - 6^2)]$

$= \pi[8^2 \times 2 + 6^2 \times 2 + (8^2 - 6^2)]$

$= \pi[128 + 72 + 28]$

$= 228\pi$ cm^2

$= 228 \times 3.14$

$= 715.92$ cm^2

Total cost of painting = Rs.715.92 × 5 Rs.3579.60

SECTION-E

36. (i) Coordinates of the position where Anjali sit = $\left(\frac{3+9}{2}, \frac{4+4}{2}\right)$ = (6, 4)

(ii) Distance between Sita and Anita

$= \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{3^2 + 3^2}$

$= 3\sqrt{2}$

(iii) Distance between Sita and Gita

$= \sqrt{(6-3)^2 + (7-4)^2}$

$= \sqrt{3^2 + 3^2}$

$= 3\sqrt{2}$

Distance between Gita and Rita = $\sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{3^2 + 3^2}$

$= 3\sqrt{2}$

Gita is equidistant from Sita and Rita

OR

$AB = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$BC = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$CD = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$DA = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$AC = \sqrt{6^2} = 6$

$BD = \sqrt{6^2} = 6$

Thus, AB = BC = CD = DA

AC = BD

⇒ ABCD is a square.

37.

Time (in sec)	x	f	cf	fx
0-20	10	8	8	80
20-40	30	10	18	300
40-60	50	13	31	650
60-80	70	6	37	420
80-100	90	3	40	270
Total		40		1720

(i) Mean = $\frac{1720}{40}$ = 43

OR

Median class = 40 – 60

Modal class = 40 – 60

Therefore, the sum of the lower limits of median and modal class = 40 + 40 = 80

(ii) Number of students who finished the race within 1 minute = 8 + 10 + 13 = 31

(iii) Number of students who finished the race within 40 sec = 8 + 10 = 18.

38. (i) We have, speed of the stream be x km/h

Speed of a motor boat is 20 km/h

So the speed of motorboat in upstream will be (20 – x) km/h.

The speed will be less in upstream journey.

(ii) Speed = $\dfrac{\text{distance}}{\text{time}}$

Speed is the distance travelled per unit time.

(iii) Let speed of the stream be x km/h

For covering the distance of 15 km the boat took one hour more for upstream than downstream.

We have,

$$\Rightarrow \frac{15}{(20-x)} - \frac{15}{(20+x)} = 1$$

On simplifying we get,

$x^2 + 30x - 400 = 0$

OR

On solving the quadratic equation $x^2 + 30x - 400 = 0$

We get, x = 10 and –40(rejected)

So the speed of the current is 10km/h.

MATHEMATICS SAMPLE PAPER # 2

ANSWER AND SOLUTIONS

SECTION-A

1. Option (1)

$x = -\frac{b}{a}$

2. Option (4)

No solution

3. Option (2)

3 units

4. Option (2)

$\frac{3}{4}$

5. Option (2)

2

6. Option (2)

7

7. Option (3)

7.8

8. Option (4)

$\frac{1}{2}$

9. Option (4)

–1

10. Option (2)

6

11. Option (1)

$\frac{17}{32}$

12. Option (3)

$15\sqrt{3}\,\text{m}$

13. Option (2)

50°

14. Option (1)

x = 2, y = 3

15. Option (3)

79°

16. Option (1)

7 cm

17. Option (2)

Similar but not congruent.

18. Option (1)

$\frac{5}{2}$

19. Option (2)

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Option (4)

Assertion (A) is false but Reason (R) is true.

SECTION-B

21. Area of large circle = $\pi(7)^2$

Area of small circle = $\pi\left(\frac{7}{2}\right)^2$

Required area of sector = area of large sector – area of small sector

$= \frac{30°}{360°} \times \pi\left[7^2 - \left(\frac{7}{2}\right)^2\right]$

$= \frac{1}{12} \times \pi \times \left(49 - \frac{49}{4}\right)$

= 9.625 cm^2

OR

Distance covered in 1 revolution = circmference of wheel = πd

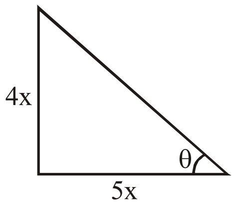
= $\pi \times 1.26$ m

Distance covered in 500 revolutions

$= 500 \times \frac{22}{7} \times 1.26$

= 1980 m = 1.98 km

22. $\tan\theta = \frac{4}{5} \Rightarrow \tan\theta = \frac{P}{B}$



$= \frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$

divided by cosθ

$= \frac{5\tan\theta - 3}{5\tan\theta + 3} = \frac{5\times\frac{4}{5} - 3}{5\times\frac{4}{5} + 3} = \frac{4-3}{4+3} = \frac{1}{7}$

OR

$\cos A = 1 - \cos^2 A$

$\cos A = \sin^2 A$ ......(1)

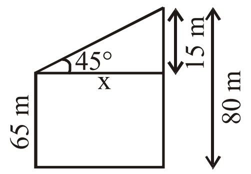
$\sin^2 A + \sin^4 A = \sin^2 A(1 + \sin^2 A)$

$= \cos A(1 + \cos A)$

$= \cos A + \cos^2 A$

$= 1$ [Given]

23.



$\tan 45^\circ = \frac{15}{x}$

x = 15 m

24. Let us assume $(5 + 2\sqrt{3})$ is a rational number.

$\therefore\ 5 + 2\sqrt{3} = \frac{p}{q}$

(where, q ≠ 0 and p and q are co-prime integers)

$\Rightarrow \sqrt{3} = \frac{p - 5q}{2q}$

This contradicts the given fact that $\sqrt{3}$ is irrational.

Hence, $(5 + 2\sqrt{3})$ is an irrational number.

25. Total English alphabets = 26

Number of consonants = 21

$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}}$

$P(E) = \frac{21}{26}$

SECTION-C

26. $\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\times h$

Class	Frequency	
0 – 10	7	
10 – 20	10	f_0
20 – 30	15	f_1
30 – 40	8	f_2
40 – 50	10	

Modal class = 20 – 30

$f_0 = 10$ $\ell = 20$

$f_1 = 15$ $h = 10$

$f_2 = 8$

$\text{Mode} = 20 + \left(\frac{15 - 10}{2\times15 - 10 - 8}\right)\times 10$

$= 20 + \left(\frac{5}{12}\right)\times 10$

$= 20 + \frac{25}{6}$

= 20 + 4.16 = 24.16

27. Opposite sides of rectangle are equal

3x + y = 7 .... (1) x + y = 5 ...(2)

$$\begin{array}{r} 3x + y = 7 \\ \underline{x + y = 5} \\ 2x = 2 \end{array}$$

x = 1

x + y = 5

1 + y = 5

y = 5 – 1 = 4

x = 1, y = 4

OR

Let the cost price of 1 bat is ₹ x

and the cost price of 1 ball is ₹ y

$7x + 6y = 3800$ ....(1)

$3x + 5y = 1750$ .....(2)

From (i)

$7x = 3800 - 6y$

$x = \frac{3800-6y}{7}$ ....(3)

Substituting value of x from (3) in (2), we get

$3\left(\frac{3800-6y}{7}\right) + 5y = 1750$

$11400 - 18y + 35y = 12250$

$17y = 850$

$y = 50$

From (3) $x = \frac{3800-300}{7} = 500$

Thus, cost price of 1 bat is ₹ 500 and 1 ball is ₹ 50

28. Let $\sqrt{2}$ = rational number

$\sqrt{2} = \frac{a}{b}$ (a & b are co-prime number)

$\sqrt{2}b = a$

By squaring both the sides

$2b^2 = a^2$ .... (1)

So, a^2 is multiple of 2

a is multiple of 2

$a = 2c$ (c is some integer)

Squaring both sides

$a^2 = 4c^2$ $\because a^2 = 2b^2$

$2b^2 = 4c^2$

$b^2 = 2c^2$ .....(2)

b^2 is multiple of 2

b is multiple of 2

From equation (1) & (2)

a & b are multiple of 2

therefore a & b are not co-prime

This contradict the fact that a & b have no common factor other than 1.

This contradiction arises by assuming that $\sqrt{2}$ is rational.

Hence, $\sqrt{2}$ is irrational number.

29. $7a_7 = 11a_{11}$

$7(a + 6d) = 11(a + 10d)$

$7a + 42d = 11a + 110d$

$-4a = +68d$

$a = -17d$

$a_{18} = a + 17d$

$= -17d + 17d = 0$

OR

$a = 3,\ d = 8 - 3 = 5,\ \ell = 253$

$a_{20} = \ell - (n - 1)d$

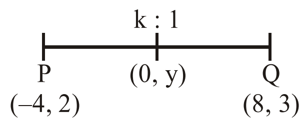
$= 253 - (20 - 1)5$

$= 253 - 19 \times 5$

$= 253 - 95$

$= 158$

30.

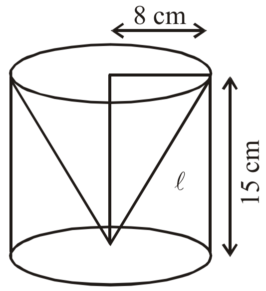


$\frac{8k-4}{k+1} = 0$

$\Rightarrow 8k - 4 = 0$

$k = 1 : 2$

31.



Here, h = 15 cm, r = 8 cm

$\ell = \sqrt{15^2 + 8^2} = 17$ cm

Total S.A. $= 2\pi rh + \pi r^2 + \pi r\ell = \pi r(2h + r + \ell)$

$= \frac{22}{7} \times 8[30 + 8 + 17]\ cm^2$

$= \frac{22}{7} \times 8 \times 55\ cm^2$

$= 1383\ cm^2$ (Approx)

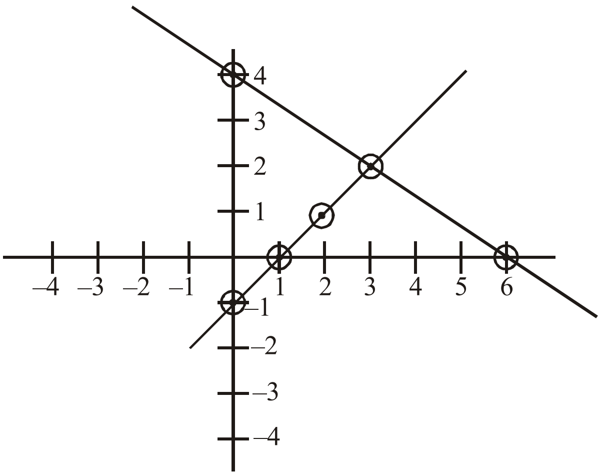
SECTION-D

32. $2x + 3y = 12$

x	3	0	6
y	2	4	0

$x - y = 1$

x	1	0	2
y	0	–1	1



(3, 2)

OR

Let speed of stream be x km/hr

$$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2} = \frac{9}{2}$$

$$\frac{30[15+x+15-x]}{225-x^2} = \frac{9}{2}$$

$200 = 225 - x^2$

$x^2 = 25$

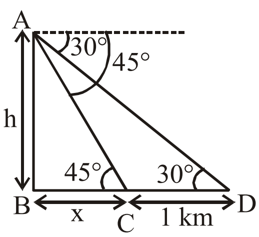
$x = 5$

Thus, speed of stream is 5 km/hr.

33. Let AB → height of hill(h)

In ΔABC

$$\tan 45° = \frac{AB}{BC}$$



$$1 = \frac{h}{x}$$

$\Rightarrow h = x$ ....(1)

In ΔABD

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+1}$$

Using $\frac{1}{\sqrt{3}} = \frac{h}{h+1}$

$h + 1 = \sqrt{3}h$

$1 = (\sqrt{3} - 1)h$

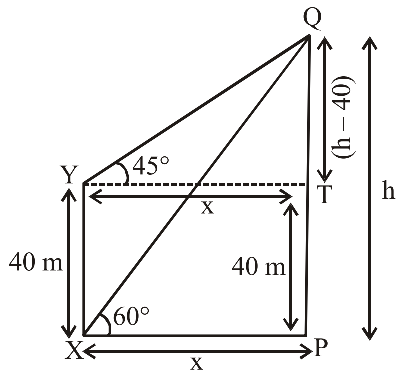
$$h = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$h = \frac{\sqrt{3}+1}{2} = \frac{1.732+1}{2}$$

$$= \frac{2.732}{2}$$

$h = 1.366$ km

OR



Let PX = x m and PQ = h m

QT = (h – 40) m

In ΔPQX,

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$\Rightarrow h = \sqrt{3}x$ .....(i)

In ΔQTY, $\tan 45° = \frac{h-40}{x}$

$$\Rightarrow 1 = \frac{h-40}{x}$$

$\Rightarrow x = h - 40$ ....(ii)

Solving (i) and (ii), $x = \sqrt{3}x - 40$

$(\sqrt{3}x - x) = 40$

or $(\sqrt{3} - 1)x = 40$

or $x = \dfrac{40}{\sqrt{3}-1} = 20\left(\sqrt{3}+1\right)m$ = 54.64 cm

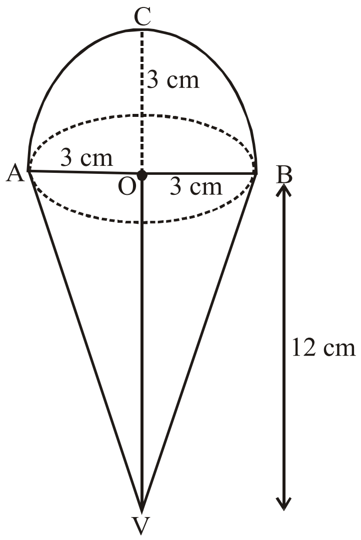
$h = \sqrt{3} \times 20(\sqrt{3}+1) = 20(3+\sqrt{3})m$

= 20(3 + 1.73) = 20 × 4.73

Hence, the height of tower is 94.6 m

34. We have,

Volume of ice-cream in the container shaped like a right circular cylinder having radius 6 cm and height 15 cm



$= \pi \times 6^2 \times 15$ cm³

Volume of one ice-cream cone shown in figure

$= \left\{\dfrac{2}{3}\pi \times 3^3 + \dfrac{1}{3}\pi \times 3^2 \times 12\right\}$ cm²

= (18π + 36π) cm³ = 54π cm³

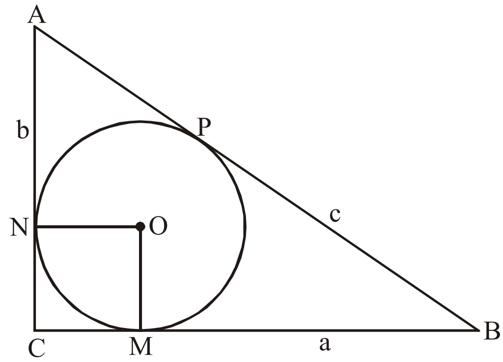
Let the total number of cones that can be filled with the ice-cream given in the container be n. Then,

Volume of ice-cream in n cones = Volume of ice-cream in the container

$\Rightarrow 54\pi \times n = \pi \times 36 \times 15$

$\Rightarrow n = \dfrac{\pi \times 36 \times 15}{54\pi} = 10$

35. Let circle touches CB at M, CA at N and AB at P. Now, OM ⊥ CB and ON ⊥ AC



(radius ⊥ tangent)

OM = ON (radii)

CM = CN (Tangents)

∴ OMCN is a square.

Let OM = r = CM = CN

AN = AP, CN = CM and BM = BP

(tangent from external point)

AN = AP

⇒ AC – CN = AB – BP

b – r = c – BM

b – r = c – (a – r)

b – r = c – a + r

∴ 2r = a + b – c

$r = \dfrac{a+b-c}{2}$

Hence proved

SECTION-E

36. (i) a = 51

d = –2

AP = 51, 49, 47.......

(ii) Goal = 31 second

n = number of days

∴ $a_n = 31$

a + (n – 1)d = 31

51 – 2n + 2 = 31

–2n = 31 – 53

–2n = –22

n = 11

(iii) $a_n = 2n + 3$(given)

$\therefore a_1 = 2 \times 1 + 3 = 5$

$a_2 = 2 \times 2 + 3 = 7$

$a_3 = 2 \times 3 + 3 = 9$

So, common difference $= a_2 - a_1$

$= 7 - 5$

$= 2$

OR

Since, 2x, x + 10, 3x + 2 are in A.P., this common difference will remain same.

$x + 10 - 2x = (3x + 2) - (x + 10)$

$10 - x = 2x - 8$

$3x = 18$

$x = 6$

37. (i) Class mark $= \dfrac{\text{Lower limit} + \text{Upper limit}}{2}$

$\Rightarrow m = \dfrac{\text{Lower limit} + b}{2}$

$\Rightarrow$ lower limit $= 2m - b$

(ii)

Class	Class-mark	Frequency (f_i)	$d_i = x_i - A$	f_id_i
150-200	175	14	–150	–2100
200-250	225	56	–100	–5600
250-300	275	60	–50	–3000
300-350	325 = A	86	0	0
350-400	375	74	50	3700
400-450	425	62	100	6200
450-500	475	48	150	7200
Total		**400**		**6400**

Average lifetime of a packet

$$= A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 325 + \frac{6400}{400} = 341 \text{hrs}$$

OR

Also, cumulative frequency for the given distribution are 14, 70, 130, 216, 290, 352, 400

$\therefore$ c.f. just greater than 200 is 216, which is corresponding to the interval 300 – 350.

$\ell = 300$, f = 86, c.f. = 130, h = 50

$\therefore$ Median

$$= \ell + \left(\frac{\frac{N}{2} - c.f_i}{f}\right) \times h = 300 + \left(\frac{200 - 130}{86}\right) \times 50$$

$= 300 + 40.697 = 340.697 = 341$ hrs

(iii) We know that,

Mode = 3 Median – 2 Mean

$= 3(341) - 2(341) = 341$ hrs

38. (i) As they are tangents from an external point to a circle. They are equal to each other. SK = SC

(ii) $OR^2 = OK^2 - KR^2 = 5^2 - 4^2 = 3^2$

OR = 3 m

OR

$\angle SKR + \angle OKR = 90°$

[The radius to the point of contact of tangent is perpendicular to tangents]

(iii) Let SR = x

In ΔSKR

$SK^2 = 8^2 + x^2$ ....(1)

In ΔSKO

$(x + 4)^2 = 5^2 + SK^2$

$SK^2 = (x + 4)^2 - 5^2$ ....(2)

From (1) and (2)

$x^2 + 64 = x^2 + 16 + 8x - 25$

$8x = 64 - 16 + 25$

$8x = 73$

$x = \dfrac{73}{8} = 9.125$ m

MATHEMATICS — SAMPLE PAPER # 3

ANSWER AND SOLUTIONS

SECTION-A

1. Option (3)

28

2. Option (2)

9696

3. Option (3)

$x+\frac{1}{x}$ is not a polynomial

4. Option (2)

7

5. Option (2)

$\frac{1}{3}$

6. Option (1)

10

7. Option (4)

135°

8. Option (3)

p(p + 1)

9. Option (2)

> 0

10. Option (1)

unique

11. Option (3)

5 units

12. Option (4)

$\frac{1+\sqrt{3}}{2\sqrt{2}}$

13. Option (2)

$40\sqrt{3}\,m$

14. Option (4)

$x^2 - 2x - 15$

15. Option (2)

Rs.13

16. Option (3)

$BD \times CD = AD^2$

17. Option (4)

2

18. Option (3)

50°

19. Option (2)

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Option (4)

Assertion is false but reason is true.

SECTION-B

21. r = 0.2 m

One revolution = $2\pi r = 2 \times \frac{22}{7} \times 0.2$ m

Number of revolutions = $\frac{176}{2\times\frac{22}{7}\times0.2} = 140$

22.

$\sqrt{(10-2)^2+(y+3)^2}=10$

$\Rightarrow 8^2 + (y + 3)^2 = 100$

$\Rightarrow (y + 3)^2 = 36$

$\Rightarrow y + 3 = \pm6$

$\Rightarrow y + 3 = 6 \;;\; y + 3 = -6$

$\Rightarrow y = 3 \;;\; y = -9$

23. Total numbers = 25

Favourable numbers = 2,3,5,7,11,13,17,19,23 = 9

$P(E) = \frac{9}{25}$

24. $1+\sin^2\theta = 3\sin\theta\cos\theta$

Divided by $\cos^2\theta$

$\sec^2\theta + \tan^2\theta = 3\tan\theta$

$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$

$2\tan^2\theta - 3\tan\theta + 1 = 0$

$2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$(2\tan\theta - 1)(\tan\theta - 1) = 0$

$\tan\theta = \frac{1}{2}$ or $\tan\theta = 1$

Hence proved

OR

$\cot^2\theta - \frac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta = -1$

25. $a = 3,\ d = 15 - 3 = 12$

$a_n = 132 + a_{54}$

$a + (n - 1)d = 132 + a + 53d$

$3 + (n - 1)12 = 132 + 3 + 53 \times 12$

$\Rightarrow (n - 1)12 = 132 + 53 \times 12$

$\Rightarrow (n - 1)12 = 768$

$n - 1 = 64$

$n = 65$

OR

ar.Δalong y-axis $= \frac{1}{2} \times 2 \times 8 = 8$ sq. units

ar.Δalong x-axis $= \frac{1}{2} \times 2 \times 2 = 2$ sq. units

$\Rightarrow$ Ratio $= \frac{2}{8} = 1 : 4$

SECTION-C

26. Two solutions of each linear equation

$x + 3y = 6$ ...(i)

and $2x - 3y = 12$ ...(ii)

are given below.

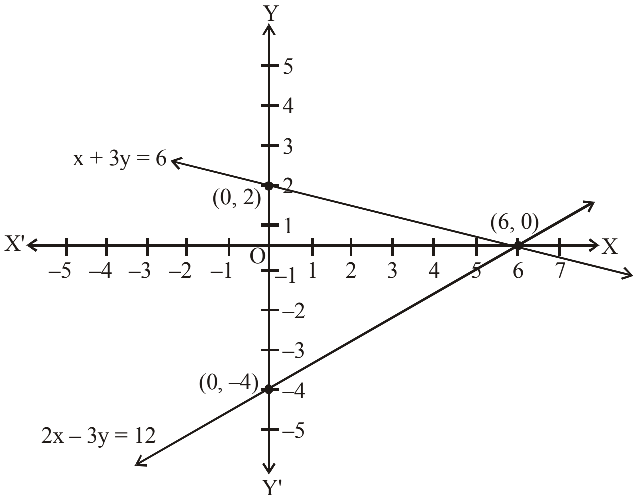
(i)

x	6	0
y	0	2

(ii)

x	6	0
y	0	– 4

The graphical representation of the given pair of linear equations is as follows :



Thus, the coordinates of point where the line $x + 3y = 6$ intersects the y-axis at (0, 2) and the line $2x - 3y = 12$ intersects the y-axis at (0, – 4).

OR

Let the fraction be $\frac{x}{y}$.

According to question

$\therefore x + y = 2x + 4 \Rightarrow x = y - 4$ ...(1)

Also, $\frac{x+3}{y+3} = \frac{2}{3}$ ....(2)

$\Rightarrow \frac{y-4+3}{y+3} = \frac{2}{3}$

$\Rightarrow \frac{y-1}{y+3} = \frac{2}{3}$

$\Rightarrow 3y - 3 = 2y + 6 \Rightarrow y = 9$

Substituting the value of y in (i), we get

$x = 5$

Thus, the required fraction is $\frac{5}{9}$.

27. 

Since, given line is divided by y-axis. Hence, its abscissa is 0.

$$\frac{-k+5}{k+1}=0 \Rightarrow k=5$$

Thus, y-axis divides the line is the ratio 5 : 1.

$$\text{Coordinate} = \left(\frac{-5+5}{6},\frac{-20-6}{6}\right)=\left(0,-\frac{13}{3}\right)$$

28.

x	y	xy
3	6	18
5	8	40
7	15	105
9	p	9p
11	8	88
13	4	52

$\bar{x}=7.5$

$$7.5=\frac{303+9p}{41+p}$$

307.5 + 7.5p = 303 + 9p

4.5 = 1.5 p

p = 3

OR

We observe that the class 12 – 15 has maximum frequency. Therefore, this is the modal class.

We have,

$\ell = 12$, $h = 3$, $f = 23$, $f_1 = 10$ and $f_2 = 21$

$$\therefore \text{Mode} = \ell + \frac{f-f_1}{2f-f_1-f_2}\times h$$

$$\Rightarrow \text{Mode} = 12 + \frac{23-10}{46-10-21}\times 3$$

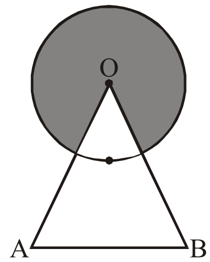
$$\Rightarrow \text{Mode} = 12 + \frac{13}{15}\times 3 = 12 + \frac{13}{5} = 14.6$$

29. Sum of the zeros = $\frac{1}{4}$ × (product of zeros)

$$k+3=\frac{1}{4}(5k-3)$$

4k + 12 = 5k – 3

15 = k

30. 

$\angle AOB = 60°$

Area of shaded region = Area of major sector

$$=\frac{300°}{360°}\times\frac{22}{7}\times(6)^2 = \frac{5}{6}\times\frac{22}{7}\times 6\times 6$$

$= 94.28 \text{ cm}^2$

31. We have,

$$\text{LHS} = \frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$\Rightarrow \text{LHS} = \frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

$$\Rightarrow \text{LHS} = \frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$\Rightarrow \text{LHS} = \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

[$\because \sin^2\theta + \cos^2\theta = 1$]

$$\Rightarrow \text{LHS} = \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

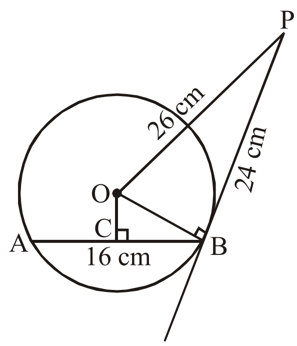
$$\Rightarrow \text{LHS} = \frac{2}{\sin\theta} = 2\text{cosec}\theta = \text{RHS}$$

SECTION-D

32. Given, AB is a chord of circle with centre O and tangent PB = 24 cm, OP = 26 cm.

Construction : Join O to B and draw OC ⊥ AB.

By Pythagoras theorem,



$OB = \sqrt{(26)^2 - (24)^2}$

$= \sqrt{676 - 576} = \sqrt{100}$

= 10 cm

Now, in ΔOBC, $BC = \frac{1}{2}AB = \frac{16}{2} = 8$ cm

(Perpendicular drawn from the centre to a chord bisects it.)

OB = 10 cm

$OC^2 = OB^2 - BC^2$

$= 10^2 - 8^2$

$OC^2 = 36$

OC = 6 cm

∴ Distance of the chord from the centre = 6 cm

33.

C.I	x_i	u_i	f_i	f_iu_i
35 – 40	37.5	–5	1	–5
40 – 45	42.5	–4	2	–8
45 – 50	47.5	–3	3	–9
50 – 55	52.5	–2	x	–2x
55 – 60	57.5	–1	y	–y
60 – 65	62.5 = A	0	6	0
65 – 70	67.5	1	8	8
70 – 75	72.5	2	4	8
75 – 80	77.5	3	2	6
80 – 85	82.5	4	3	12
85 – 90	87.5	5	2	10
Total			$\Sigma f_i = 31 + x + y$	$\Sigma f_iu_i = 22 - 2x - y$

Here, $\Sigma f_i = 31 + x + y = 40$

$\Rightarrow \quad x + y = 9$

$\Sigma f_iu_i = 22 - 2x - y$

∴ Mean $= A + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$

$\Rightarrow \quad 63.5 = 62.5 + \frac{(22 - 2x - y)}{40} \times 5$

$\Rightarrow \quad 2x + y = 14$

Solving equations (i) and (ii),

x = 5 and y = 4.

OR

Height	Frequency	c.f.
100 – 120	12	12
120 – 140	14	26
140 – 160	8	34
160 – 180	6	40
180 – 200	10	50
Total	**50**	

Here, $N = 50 \Rightarrow \frac{N}{2} = \frac{50}{2} = 25$

So, Median class = 120 – 140

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

$$= 120 + \left(\frac{25 - 12}{14}\right) \times 20$$

$$= 120 + \frac{260}{14}$$

= 120 + 18.57

Median = 138.57

34. Let BC be building of height 20 m and CD be the tower of height h m.

Let A be point on the ground at a distance of x m from the foot of the building.

In right ΔABC,

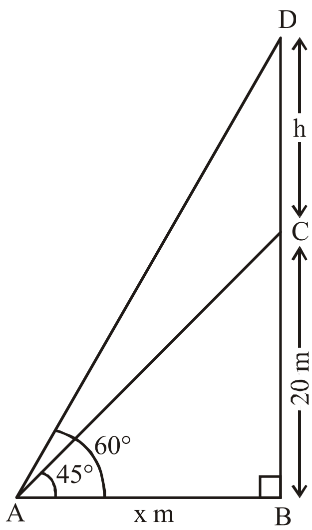
$\tan 45° = \frac{BC}{AB}$

$\Rightarrow \quad 1 = \frac{20}{x}$

$\Rightarrow \quad x = 20$ m ...(i)

In right ΔABD,

$\tan 60° = \frac{BD}{AB}$



$\Rightarrow \quad \sqrt{3} = \dfrac{h+20}{x}$

$\Rightarrow \quad \sqrt{3} = \dfrac{h+20}{20}$ ...(ii)

$\Rightarrow \quad h = 20\sqrt{3} - 20$

$= 20(\sqrt{3} - 1)$

$= 20 \times 0.732$

$= 14.64$ m

Height of tower = 14.64 m

OR

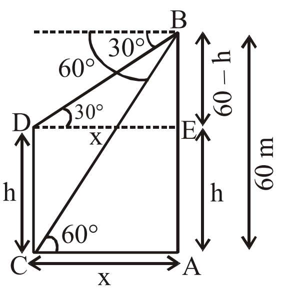
Let AB be the building and CD be the tower. Let CD = h metres. It is given that from the top of the building B, the angles of depression of the top D and the bottom C of the tower CD are 30° and 60° respectively.

$\therefore \angle EDB = 30°$ and $\angle ACB = 60°$

Let AC = DE = x

In ΔDEB, right angled at E,

We have



$\tan 30° = \dfrac{BE}{DE}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{60-h}{x}$

$\Rightarrow x = \sqrt{3}(60 - h)$ ....(1)

In ΔCAB, right-angled at A, we have

$\tan 60° = \dfrac{AB}{CA}$

$\Rightarrow \sqrt{3} = \dfrac{60}{x} \Rightarrow x = \dfrac{60}{\sqrt{3}} = \dfrac{60\sqrt{3}}{3} = 20\sqrt{3}$

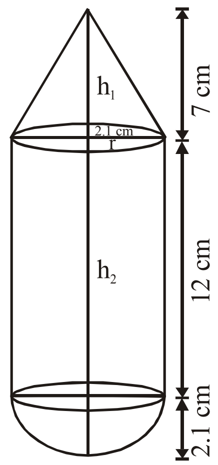
Putting the value of x in (1), we get

$20\sqrt{3} = \sqrt{3}(60 - h)$

$\Rightarrow 20 = 60 - h \Rightarrow h = 60 - 20 = 40$ metres

Thus, the height of the tower is 40 metres.

35.



Height of the conical portion = 7 cm

Diameter of the conical portion = 4.2 cm

Volume of the conical portion = $\dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times (2.1)^2 \times 7$

$= 32.34 \text{ cm}^3$ ...(1)

Height of the cylinder = 12 cm

Radius of the cylinder = 2.1 cm

Volume of the cylinder

$= \pi r^2 h = \dfrac{22}{7} \times (2.1)^2 \times 12$

$= 166.32 \text{ cm}^3$ ...(2)

Volume of the hemisphere

$= \dfrac{2}{3}\pi r^3 = \dfrac{2}{3} \times \dfrac{22}{7} \times (2.1)^3 = 19.40 \text{ cm}^3$ ...(3)

On adding (1),(2) and (3), we get

Volume of the solid toy = Volume of cone + Volume of cylinder + Volume of hemisphere

$= 32.34 + 166.32 + 19.40 = 218.06 \text{ cm}^3$

SECTION-E

36. (i) According to given situation, we have

$x + 10y = 75$ ...(i)

$x + 15y = 110$ ...(ii)

(ii) So let fixed charge is x.

then $x + 8y = 91$ ...(i)

$x + 14y = 145$ ...(ii)

Solving (i) and (ii),

$x = 19$

$y = 9$

30 km travelling charge = $x + 30y$

$= 19 + 30 \times 9$

= Rs.289

(iii) Solving two equations,

$x + 10y = 75$

$x + 15y = 110$

$- \quad - \quad -$

$-5y = -35$

$y = 7$

Now, putting y = 7 in equation (i)

$x + 10 \times 7 = 75$

$x + 70 = 75$

$x = 75 - 70$

$x = 5$

Now, if a person travels a distance of 50 km then, amount $= x + 50y$

$= 5 + 50 \times 7$

$= 5 + 350$

= Rs.355

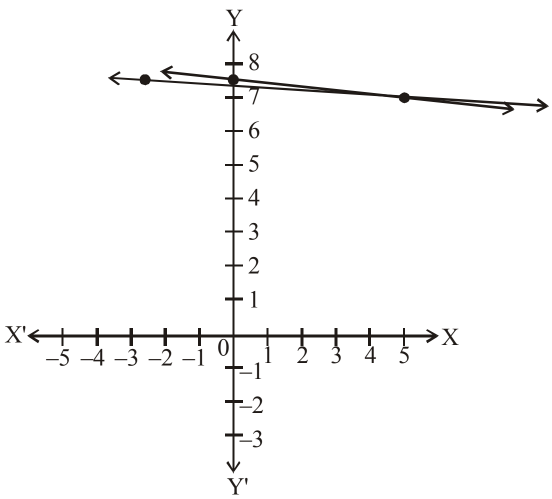
OR

$x + 10y = 75$

x	5	0
y	7	7.5

$x + 15y = 110$

x	5	–2.5
y	7	7.5



37. Number of rose plants = 135

Number of marigold plants = 225

(i) The maximum number of columns in which they can be planted = HCF of 135 and 225

$\therefore$ Prime factors of $135 = 3 \times 3 \times 3 \times 5$

and $225 = 3 \times 3 \times 5 \times 5$

$\therefore$ HCF of 135 and $225 = 3 \times 3 \times 5 = 45$

(ii) Total number of plants 135 + 225

= 360 plants

(iii) From (i) the maximum number of columns = 45

So, prime factors of $45 = 3 \times 3 \times 5$

$= 3^2 \times 5^1$

$\therefore$ Sum of exponents = 2 + 1 = 3

OR

From (ii) the total number of plants

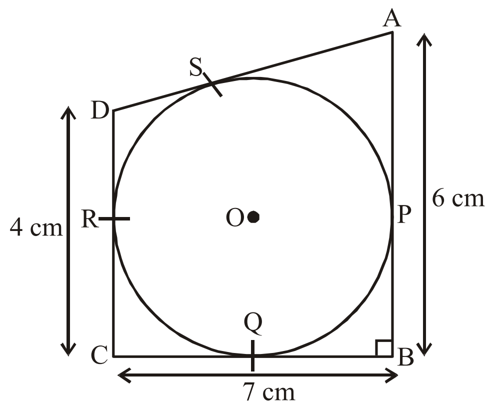
= 360

Prime factors of

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5^1$

$\therefore$ Sum of exponents = 3 + 2 + 1 = 6

38.



(i) Let CQ = x = CR

BQ = 7 – x = BP

AP = 6 – 7 + x = x – 1 = AS

DR = 4 – x = DS

AD = AS + DS = x – 1 + 4 – x = 3 cm

(ii) If CQ = 2

PB = 7 – 2 = 5 cm

(iii) If CQ = 2

DS = 4 – 2 = 2 cm

OR

Perimeter of playground = 7 + 6 + 3 + 4

= 20 cm

MATHEMATICS SAMPLE PAPER # 4

ANSWER AND SOLUTIONS

SECTION-A

1. Option (4)

More than 3

2. Option (4)

No solution

3. Option (2)

98

4. Option (2)

1 : 2

5. Option (3)

1

6. Option (1)

40°

7. Option (2)

14k

8. Option (1)

2 units

9. Option (2)

360 cm^2

10. Option (3)

$x^2 - 4x + 5$

11. Option (4)

84

12. Option (2)

1

13. Option (3)

Real and unequal

14. Option (3)

$\left(\frac{3}{2}, 2\right)$

15. Option (2)

3 cm

16. Option (2)

–1

17. Option (1)

25°

18. Option (2)

± 1

19. Option (2)

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Option (1)

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION-B

21. In ΔOQP

$\angle POR = \angle OQP + \angle OPQ$ (Exterior angle)

$\angle OPQ = \angle POR - \angle OQP$

$= 120° - 90°$

$= 30°$

22. Using the factor tree for the prime factorisation of 90 and 144, we have

$90 = 2 \times 3^2 \times 5$ and $144 = 2^4 \times 3^2$

To find the HCF, we list the common prime factor and their smallest exponents in 90 and 144 as under :

Common prime factors	Least exponents
2	1
3	2

$\therefore$ HCF $= 2^1 \times 3^2 = 2 \times 9 = 18$

To find the LCM, we list all prime factors of 90 and 144 and their greatest exponents as follows:

Prime factors of 90 and 144	Greatest exponents
2	4
3	2
5	1

$\therefore$ LCM = $2^4 \times 3^2 \times 5^1 = 16 \times 9 \times 5 = 720$

OR

Product of two numbers = 4107

HCF = 37

LCM = 111

23. $a = 3, d = 8 - 3 = 5$

$78 = a + (n - 1)d$

$78 = 3 + (n - 1)5$

$\frac{75}{5} = n - 1$

$15 = n - 1$

$n = 16$

OR

1,3,5,7, ...,49

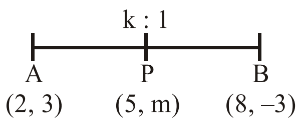
$a = 1, d = 2, n = 25$

$s_{25} = \frac{25}{2}[2 \times 1 + (25 - 1)2]$

$= \frac{25}{2}[2 + 24 \times 2]$

$= \frac{25}{2} \times 2[25] = 625$

24.



$\frac{8k+2}{k+1} = 5$

$8k + 2 = 5k + 5$

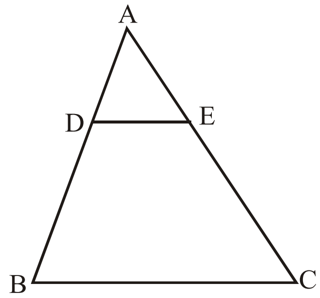
$3k = 3$

$k = 1$

$k : 1 = 1 : 1$

$m = \frac{3-3}{2} = 0$

25. We have,



BD = 4.2 cm, AD = 1.4 cm, EC = 5.4 cm and AE = 1.8 cm.

Now, $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ and $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

Thus, DE divides sides AB and AC of ΔABC in the same ratio, Therefore, by the converse of Basic Proportionality Theorem, we have

$DE \parallel BC$

SECTION-C

26. Let the radius of the protractor be r cm. Then, perimeter = 108 cm

$\Rightarrow \frac{1}{2}(2\pi r) + 2r = 108$

[$\because$ Perimeter of semi-circle = $\frac{1}{2}(2\pi r) = \pi r$]

$\Rightarrow \pi r + 2r = 108 \Rightarrow \frac{22}{7} \times r + 2r = 108$

$\Rightarrow 36r = 108 \times 7$

$\Rightarrow r = 3 \times 7 = 21$ cm

$\therefore$ Diameter of the protractor = 2r

= (2×21) cm = 42 cm

27. LHS

$= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}+1$$

= secθcosecθ + 1

= RHS

28. In ΔABD and ΔCEF, given

AB = AC

Then, ∠ABC = ∠ACB (Angles opposite to equal sides are equal)

or ∠ABD = ∠ECF

∠ADB = ∠EFC (each 90°)

∴ ΔABD ~ ΔECF (AA similarity)

Hence proved

29. Given, quadratic polynomial

$x^2 - (k + 6)x + 2(2k - 1)$

comparing it with $ax^2 + bx + c$, we get

$a = 1$, $b = -(k + 6)$, $c = 2(2k - 1)$

Since, sum of zeroes = $-\frac{b}{a} = -\left[\frac{-(k+6)}{1}\right]$

$= k + 6$

and product of zeroes = $\frac{c}{a} = \frac{2(2k-1)}{1}$

$= 2(2k - 1)$

According to question,

sum of zeroes = $\frac{1}{2}$ × product of zeroes

$\Rightarrow k + 6 = \frac{1}{2}[2(2k - 1)]$

$\Rightarrow k + 6 = 2k - 1$

$\Rightarrow k = 7$

30. Let numerator be x

denominator be y

$\frac{x+2}{y+2} = \frac{1}{3}$

$\Rightarrow 3x + 6 = y + 2$

$\Rightarrow 3x - y = -4$ ...(1)

$\frac{x+3}{y+3} = \frac{2}{5}$

$5x + 15 = 2y + 6$

$5x - 2y = -9$ ...(2)

from (1) and (2)

$6x - 2y = -8$

$5x - 2y = -9$

$-\quad +\qquad +$

$x = 1$

Put x = 1 in equation (1)

$3 - y = -4$

$y = 7$

fraction = $\frac{1}{7}$

OR

$2x + y = 6$ ...(1)

or $y = 6 - 2x$

Table for solutions for (1)

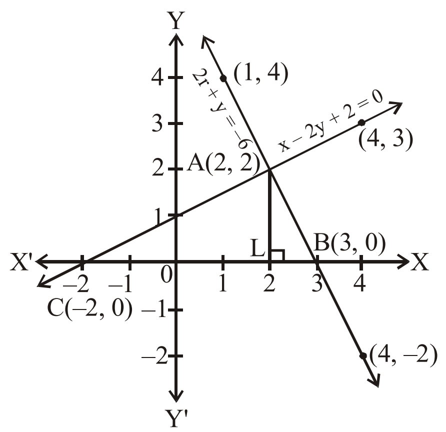
x	1	4	2
y = 6 – 2x	4	–2	2

$x - 2y + 2 = 0$ ...(2)

or $y = \frac{x+2}{2}$

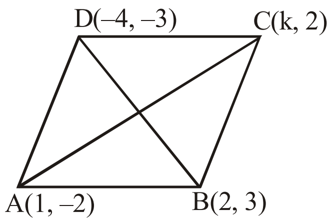
Table for solution for (2)

x	0	4	–2
$y = \frac{x+2}{2}$	1	3	0



The two straight lines intersects at A(2, 2).

31.



Diagonals of parallelogram bisect each other

$\Rightarrow$ midpoint of AC = midpoint of BD

$$\Rightarrow \left(\frac{1+k}{2}, \frac{-2+2}{2}\right) = \left(\frac{-4+2}{2}, \frac{-3+3}{2}\right)$$

$$\Rightarrow \frac{1+k}{2} = \frac{-2}{2}$$

$\Rightarrow k = -3$

OR

Let P(x, y) is equidistant from A(–5, 3) and B(7, 2)

AP = BP

$$\Rightarrow \sqrt{((x+5)^2+(y-3)^2)} = \sqrt{((x-7)^2+(y-2)^2)}$$

$\Rightarrow x^2 + 10x + 25 + y^2 - 6y + 9$

$= x^2 - 14x + 49 + y^2 - 4y + 4$

$10x - 6y + 34 = -14x - 4y + 53$

$10x + 14x - 6y + 4y = 53 - 34$

$24x - 2y = 19$

$24x - 2y - 19 = 0$

is the required relation.

SECTION-D

32.

Classes	Frequencies
100 – 150	4
150 – 200	5
200 – 250	12
250 – 300	2
300 – 350	2

Modal class $\to$ (200 – 250)

$$\text{Mode} = \ell + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 200 + \left[\frac{12-5}{2\times 12 - 5 - 2}\right] \times 50$$

$$= 200 + \frac{7}{17} \times 50$$

$$= 200 + \frac{350}{17}$$

$= 200 + 20.588$

≈ 220.59

OR

We have the following table :

Classes	Frequencies (f_i)	Class mark (x_i)	$u_i = \frac{x_i - 170}{20}$	$f_i \times u_i$
120 – 140	4	130	–2	–8
140 – 160	f	150	–1	–f
160 – 180	20	170 = a	0	0
180 – 200	12	190	1	12
200 – 220	6	210	2	12
220 – 240	8	230	3	24
Total	$n = \Sigma f_i = f + 50$			$\Sigma f_i u_i = 40 - f$

We are given that

Mean $\bar{x} = 180$

$$\Rightarrow a + h \times \left\{\frac{1}{n}\Sigma f_i u_i\right\} = 180$$

$$\Rightarrow 170 + 20 \times \left(\frac{40-f}{50+f}\right) = 180$$

$$\Rightarrow 20 \times \left(\frac{40-f}{50+f}\right) = 10$$

$\Rightarrow 2 \times (40 - f) = 50 + f$

$\Rightarrow 80 - 2f = 50 + f \Rightarrow 3f = 30 \Rightarrow f = 10$

33. Sum of first seven terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_7 = \frac{7}{2}[2a + (7-1)d] = \frac{7}{2}[2a + 6d]$$

$\Rightarrow \quad 63 = 7a + 21d$

$$\Rightarrow \quad a = \frac{63 - 21d}{7} \quad .... (1)$$

$$\Rightarrow \quad S_{14} = \frac{14}{2}[2a + 13d]$$

$\Rightarrow \quad S_{14} = 7\,[2a + 13d] = 14a + 91d$

But ATQ,

$S_{1-7} + S_{8-14} = S_{14}$

$63 + 161 = 14a + 91d$

$\Rightarrow 224 = 14a + 91d$

$2a + 13d = 32$

$2\left(\frac{63-21d}{7}\right) + 13d = 32$ (from 1)

$\Rightarrow 126 - 42d + 91d = 224$

$\Rightarrow 49d = 98$

$\Rightarrow d = 2$

$\Rightarrow a = \frac{63-21\times 2}{7} = \frac{63-42}{7} = 3$

$\Rightarrow a_{28} = a + 27d = 3 + 27 \times 2$

$\Rightarrow a_{28} = 3 + 54 = 57$

OR

Let n terms of AP give a sum of 636

Here, a = 9, d = 17 – 9 = 8

$S_n = \frac{n}{2}[2a + (n-1)d]$

$636 = \frac{n}{2}[2 \times 9 + (n-1)8]$

$636 = n[9 + (n-1)4]$

$636 = 9n + 4n^2 - 4n$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n(4n + 53) - 12(4n + 53) = 0$

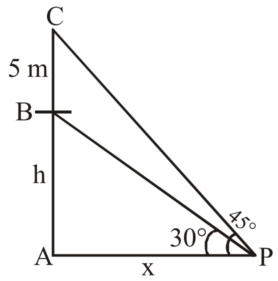
$(n - 12)(4n + 53) = 0$

$n = 12, -\frac{53}{4}$

Since, n can't be negative, $-\frac{53}{4}$ rejected

So, n = 12

34. Let AB = h be the tower and BC be the flagstaff and let AP = x



In ΔABP

$\tan 30° = \frac{h}{x}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$x = \sqrt{3}h$ ......(1)

In ΔACP

$\tan 45° = \frac{h+5}{x}$

$h + 5 = x$

$h + 5 = \sqrt{3}h$

$(\sqrt{3} - 1)h = 5$

$h = \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$h = \frac{5(\sqrt{3}+1)}{2}$

h = 6.83 m

35. Length of roof = 22 m, breadth of roof = 20 m

Let the rainfall be x cm.

Volume of water on the roof

$= \left(22 \times 20 \times \frac{x}{100}\right) m^3$

$= \frac{22x}{5} m^3$

Radius of the base of the cylindrical vessel = 1 m

Height of the cylindrical vessel = 3.5 m

Volume of water in the cylindrical vessel when it is just full $= \left(\frac{22}{7} \times 1 \times 1 \times \frac{7}{2}\right) m^3 = 11\ m^3$

$[\because V = \pi r^2 h]$

Now, volume of water on the roof = volume of water in the vessel

$\frac{22x}{5} = 11 \Rightarrow x = \left(\frac{11 \times 5}{22}\right) = 2.5$

Hence, the rainfall is 2.5 cm

SECTION-E

36. (i) Number of cards of a king of red colour = 2

Total number of cards = 52

Probability of getting a king of red colour

$= \frac{\text{Number of king of red colour}}{\text{Total number of cards}}$

$= \frac{2}{52} = \frac{1}{26}$

(ii) Number of face card = 12

Total number of cards = 52

Probability of face cards = $\frac{12}{52} = \frac{3}{13}$

(iii) Number of red face cards = 6

Total number of cards = 52

Probability of getting a red face card

$= \frac{\text{Number of red face cards}}{\text{Total number of cards}}$

$= \frac{6}{52} = \frac{3}{26}$

OR

Number of spade card = 13

Total number of cards = 52

Probability of getting a face card

$= \frac{\text{Number of spade cards}}{\text{Total number of cards}}$

$= \frac{13}{52} = \frac{1}{4}$

37. (i) $108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

(ii) Number of participants seated in each room would be HCF of all the three values above.

$60 = 2 \times 2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7$

$108 = 2 \times 2 \times 3 \times 3 \times 3$

Hence, HCF = 12

OR

Minimum number of rooms required are total number of students divided by number of students in each room.

Number of rooms = $\frac{60+84+108}{12} = 21$

(iii) $60 = 2 \times 2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7$

$108 = 2 \times 2 \times 3 \times 3 \times 3$

$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7$

$= 36 \times 15 \times 7$

$= 3780$

38. (i) Scale factor = $\frac{AC}{AE}$

$= \frac{AC}{AC+CE} = \frac{8}{8+4}$

$= \frac{8}{12} = \frac{2}{3}$

(ii) Since, $\Delta EBC \sim \Delta EFA$

$\frac{EC}{EA} = \frac{BC}{AF}$

$\Rightarrow \frac{4}{12} = \frac{3.6}{AF}$

$\Rightarrow AF = 3.6 \times 3$

= 10.8 cm

(iii) $\Delta ABC \sim \Delta ADE$

$\frac{AC}{AE} = \frac{BC}{DE}$

$\frac{8}{12} = \frac{3.6}{DE}$

$DE = \frac{3.6 \times 3}{2} = 5.4$ cm

OR

$\frac{AB}{BD} = \frac{AC}{CE}$

$\frac{AB}{BD} = \frac{8}{4}$

$\frac{AB}{BD} = \frac{2}{1}$